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THE DESIGN OF PIEZOELECTRIC TRANSDUCERS USING GOAL PROGRAMMING

Diana F. McCammon

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The design of piezoelectric transducers often involves the satisfaction of multiple, conflicting design specifications. The use of conventional mathematical programming techniques in this endeavor has been inhibited by their inability to consider these multiple objectives within their design formulations. Goal programming is an effective alternative methodology. Transucer design analysis through goal programming provides the optimal solution to the multiple criteria of various types of transducers. The solutions to five design specifications for a reasonably over page

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The design of piezoelectric transducers often involves the satisfaction of multiple, conflicting design specifications. The use of conventional mathematical programming techniques in this endeavor has been inhibited by their inability to consider these multiple objectives within their design formulations. Goal Programming is an effective alternative methodology. Transducer design analysis through goal programming provides the optimal solution to the multiple criteria of various types of transducers. The solutions to five design specifications for a reasonably complex transducer are presented. The details of the formulation and analysis of results provide an indication of the ability of this approach to resolve even more complex transducer design problems.

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A = cross-sectional area of any section perpendicular to wave motion, m^2.

$\bar{a}$ = achievement function vector.

B = susceptance, imaginary part of the admittance.

b_i = i^{th} objective goal.

c = bar velocity (effective), m/sec; or subscript for ceramic section.

CC = a form of clamped capacitance.

C_0 = capacitance of ceramic if no motion were permitted, called the clamped capacitance, picofarads.

C_{mot} = motional capacitance.

D = electric displacement.

D_y = diameter of admittance loop.

d^E_{xx} = piezoelectric constant in constant electric field.

d^T_{xx} = piezoelectric constant at constant stress.

E = electric field.

f_1 = frequency of maximum susceptance.

f_2 = frequency of minimum susceptance.

f_y = resonance frequency or frequency of maximum conductance.

f_z = frequency of minimum conductance.

G = conductance, real part of the admittance.

j = square root of minus one.

k = propagation wave number [$= k(1 - j\eta)$ to account for losses], m^{-1}.

k_{eff} = effective electromechanical coupling coefficient.

ℓ = length or dimension parallel to plane wave propagation, m.

ℓ_B = length of ceramic behind the velocity node.

ℓ_F = length of ceramic in front of the velocity node.

n_i = i^{th} negative deviation variable.

p_i = i^{th} positive deviation variable.

Q_m = mechanical quality factor.

R_0 = clamped electrical resistance, ohms.

S = strain.

s^E_{xx} = elastic constant in constant electric field.

T = stress.

t = thickness of ceramic between electrodes, m.

W_c = a function of the dimensions with the units of length, m; in equations for C_0 and ϕ, W_c depends upon geometry.

w_i = i^{th} weight factor.

$\bar{x}$ = vector of independent variables.

$\mathcal{Y}$ = electrical admittance.

$Y_i(\bar{x})$ = i^{th} objective function.

Z = impedance.

ε^T_{xx} = dielectric constant at constant stress.

η = loss factor.

ρ = density, kg/m^3.

ϕ = turns ratio of transformer from mechanical to electrical units, newtons/volt.

ψ = phase of the electrical admittance at resonance.

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CHAPTER 1

INTRODUCTION

1.1 Purpose

This study provides, through Goal Programming, a computer technique to generate suitable design parameters for piezoelectric transducers, given a set of objective criteria. The specific objectives of this thesis are:

1. To develop a mathematical model which describes the transducer and related environment and which accurately predicts a number of specific responses of the system.
2. To adapt this model to the Goal Programming formulation.
3. To investigate the behavior of Goal Programming applied to examples of "Tonpilz" design.
4. To provide a computer program to enable and encourage the use of the technique in the resolution of complex transducer design problems.

1.2 Problem Definition

One style of transducer favored for high-power underwater-sound generation is the "Tonpilz," a composition of two German words--sound and mushroom, so named for its characteristic shape as shown in Figure 1. Its particular construction is adaptable to the requirements of mounting and vibration isolation. The design of these transducers involves

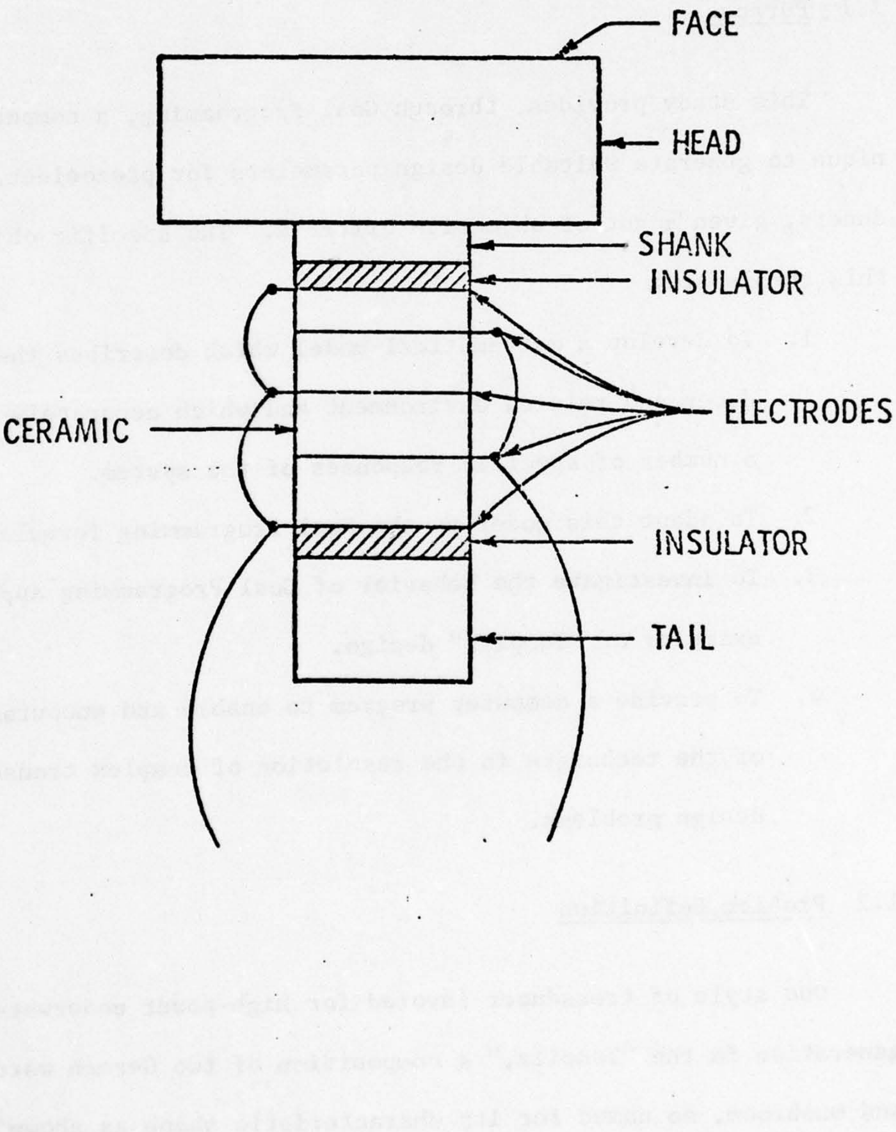


Figure 1. "Tonpilz"-Type Transducer.

typically, head and tail masses which are used for impedance matching and a central piezoceramic motor section. The actual construction of this basic three-section model sometimes requires additional parts, such as insulators and electrodes, as well as an assembly lip on the head called the "shank." Thus, a typical transducer may be comprised of different materials bonded together as illustrated in Figure 1.

Plane longitudinal stress waves passing through this conglomerate travel with different speeds within the different materials and suffer reflections at each interface. The properties of these materials must be carefully considered in the design to achieve a desired response. The length, weight, resonance frequency, mechanical quality factor, impedance, or node position may be especially important in a particular application. However, satisfaction of one requirement may adversely affect another. For example, the design of a transducer resonant at a specific frequency and of minimum weight may result in an unacceptable length or an undesirable quality factor. Thus, the design of a transducer for a specific application becomes a problem of finding an optimal compromise.

One previous approach has utilized mathematical models to predict the quality factor of and node position within a transducer at given frequencies and physical dimensions. The consideration of other properties has not been possible due to the complexity of their relations with one another. The need for a flexible, efficient, multi-objective optimization technique is clear. This need is admirably answered through Nonlinear Goal Programming (NLGP), as developed by James P. Ignizio [1]. The specific nonlinear method used is a

modification by Ignizio of a well-known method of nonlinear single-objective programming known as Pattern Search [1].

CHAPTER 2

TRANSDUCER MODEL

2.1 Transducer Theory

The piezoelectric mechanism involves sound waves passing through a crystal, exerting stress which then generates an electromotive force (EMF) as a response to this stress, and vice versa. The equations which express this interaction between stress T, strain S, electric field E, and electric displacement D are the tensor equations given by [2]:

$$S = s^{E}_{xx} \cdot T + d^{T}_{xx} \cdot E \,,$$

and (1)

$$D = d^{E}_{xx} \cdot T + \varepsilon^{T}_{xx} \cdot E \,,$$

where the tensor coefficients are:

s^{E}_{xx} = elastic constant in constant electric field,

d^{E}_{xx} and d^{T}_{xx} = piezoelectric constants in constant electric field or constant stress,

ε^{T}_{xx} = dielectric constant for constant stress,

at constant entropy.

The particular coefficients that are applicable depend upon the electrode-polarization relationship. For example, if the electrodes of a piezoceramic are perpendicular to the polarization, coefficients

s_{33}^{E}, d_{33}^{E}, d_{33}^{T}, and ε_{33}^{T} pertain. For other configurations or other types of crystals, numbers denoting other orientations may be in order [2, 9].

A method by which the system may be analyzed begins with the equivalent circuit representation for the ceramic known as "Mason's Equivalent Circuit." This circuit displayed in Figure 2 shows clearly two mechanical ports and one electrical port [3, 4]. The symbols used in Figure 2, as well as those in all subsequent figures and equations, are defined in the List of Symbols.

With W_c defined as a geometrically dependent function of length, the circuit elements of impedance Z, capacitance C_0, and transformer turns ratio ϕ can be represented as:

$$Z_1 = j\rho_c c_c A_c \tan(k_c \ell_c/2) \ , \tag{2}$$

$$Z_2 = -j\rho_c c_c A_c/\sin(k_c \ell_c) \ , \tag{3}$$

$$C_0 = W_c \ \varepsilon_{xx}^{T} \ , \tag{4}$$

and

$$\phi = d_{xx}^{T} W_c/s_{xx}^{E} \ . \tag{5}$$

This circuit can represent a stack of individual ceramic segments if the $|k\ell|$ of each segment is very small compared to π.

To represent all the remaining sections of the transducer, multiple T-sections are connected smoothly in cascade fashion to the ceramic representation at the mechanical ports. The T-sections for inert materials as shown in Figure 3 are equivalent to wave guides.

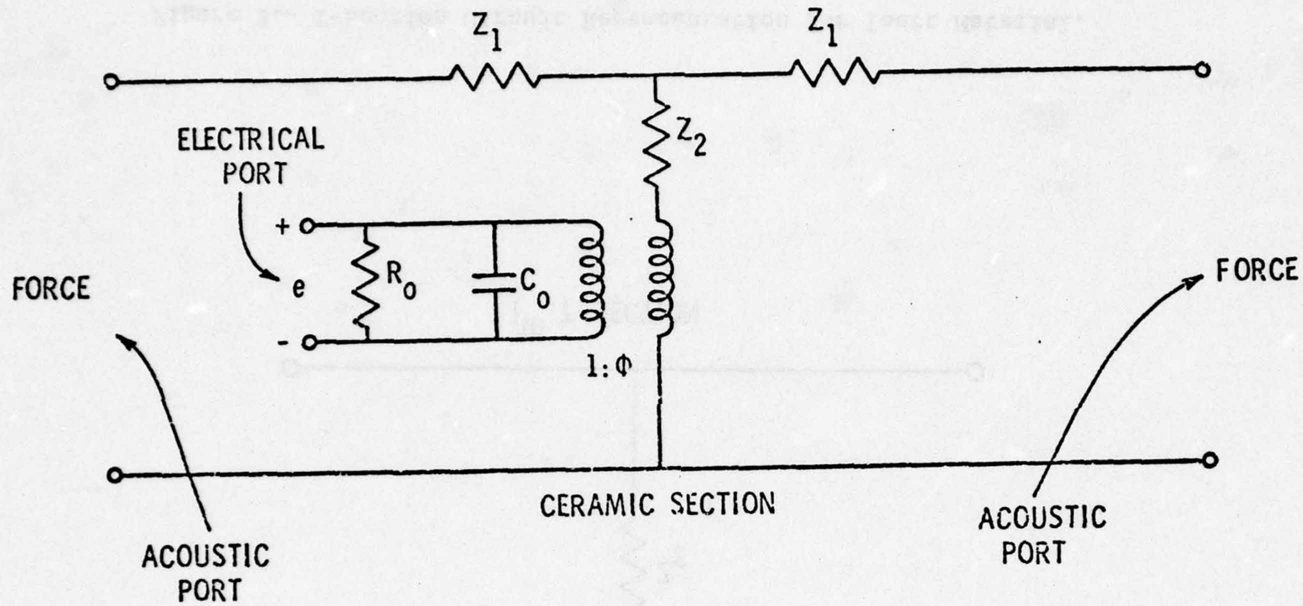


Figure 2. Mason's Equivalent Circuit for Piezoelectric Material.

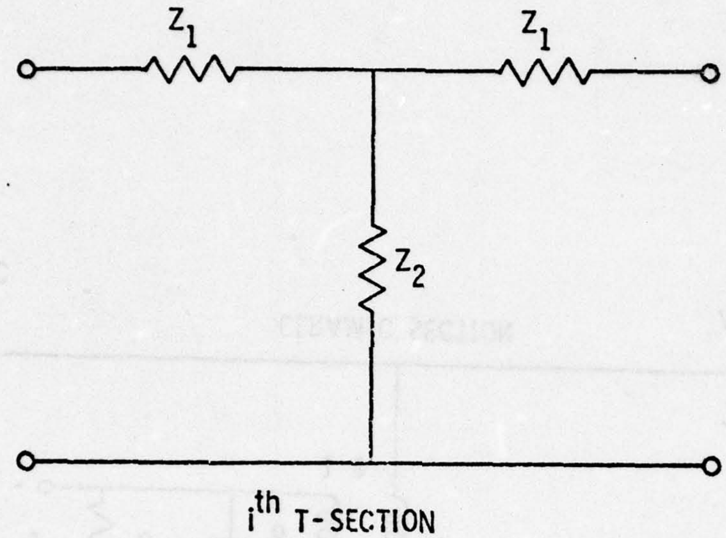


Figure 3. T-Section Circuit Representation for Inert Material.

Equations for the impedances Z_1 and Z_2 of the i^{th} T-section are given by:

$$Z_1 = j\rho_i c_i A_i \tan(k_i \ell_i/2) \tag{6}$$

and

$$Z_2 = -j\rho_i c_i A_i/\sin(k_i \ell_i) \ . \tag{7}$$

Note the identical forms of Equations (2) and (6) and of Equations (3) and (7).

On placing a load, Z_L, at one end of the T-section as shown in Figure 4, the input impedance as seen from the open terminals can be calculated using Equation (8):

$$Z_{in} = \frac{\zeta_i[Z_L + j\zeta_i \tan(k_i \ell_i)]}{[\zeta_i + jZ_L \tan(k_i \ell_i)]} \ , \tag{8}$$

where $\zeta_i = \rho_i c_i A_i$. The derivation of Equation (8) is given in Appendix A.

Use of the wave-guide equation makes it particularly easy to remove a section of the transducer if desired. By setting the length of an unwanted section equal to zero, the load is passed unaltered to the next section as can be readily seen by putting $\ell_i = 0$ in Equation (8), viz., $Z_i = Z_L$. Figure 5 depicts a typical transducer represented by Mason's Equivalent Circuit modified to include the nonceramic sections.

As a practical matter, the transducer must be mounted in a housing and also coupled to the acoustic medium (water, in this case). A common practice is to press the face of the head against an

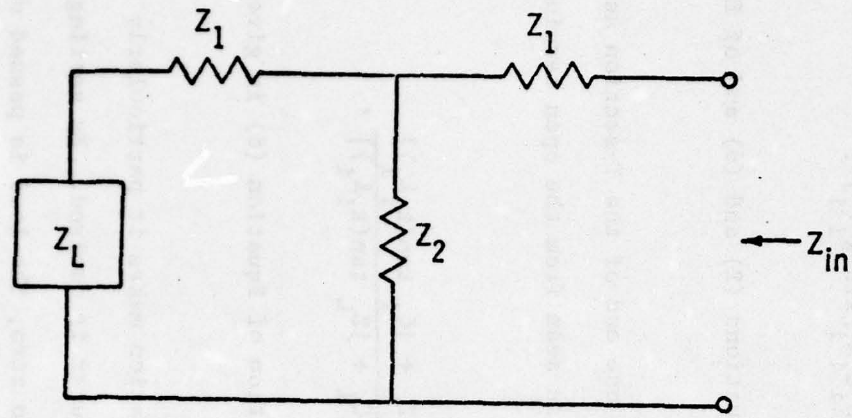
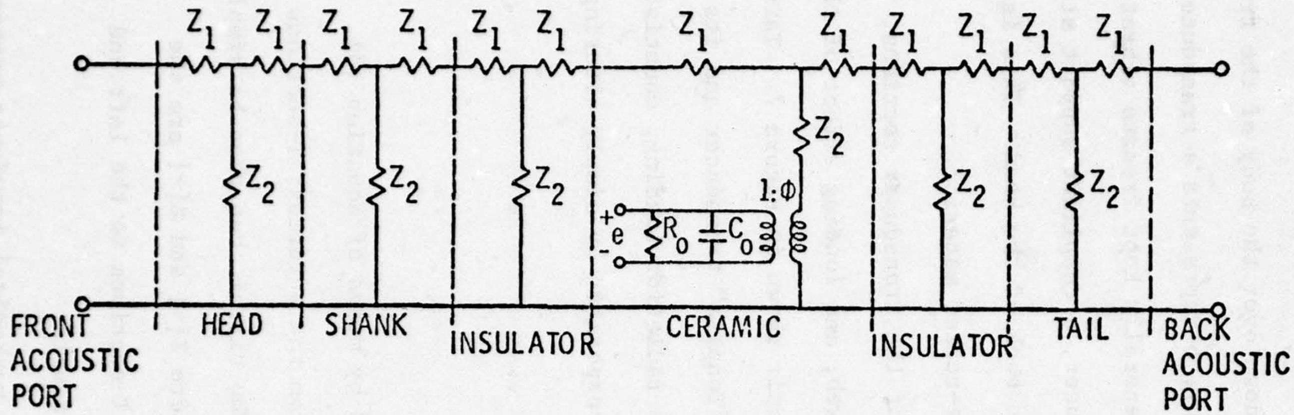


Figure 4. Input Impedance of a T-Section with Load Z_L.



ALL Z_1 OF THE FORM $Z_1 = j\rho_i c_i A_i \tan\left(\frac{k_i \ell_i}{2}\right)$

ALL Z_2 OF THE FORM $Z_2 = -j\rho_i c_i A_i / \sin(k_i \ell_i)$

Figure 5. Transducer-Equivalent Circuit with Mechanical and Electrical Ports.

acoustically transparent membrane of rubber or urethane which separates the head from the water. The pressure is applied through a plate (commonly called a web) which slips down over the body of the transducer and bolts to the housing. Figure 6 represents a transducer mounted in a housing. The tail is generally kept free to vibrate into air. For a head-mounted transducer, a compliant support structure is introduced between the web and the back of the head. This is referred to in Figure 6 as the pressure-relief material.

The entire assembly consisting of the transducer sections, rubber membrane, pressure-relief material, web, and loading effect of the medium can be represented by the circuit shown in Figure 7. This circuit is a fundamental model of a "Tonpilz" transducer and its associated housing. Any variation in radiation loading, mounting, or electrode configuration can be incorporated by simply altering this circuit representation.

2.2 Circuit Solution

The input impedance is evaluated by means of Equation (8), the wave-guide equation, beginning from the outermost T-sections inward toward the ceramic section. The circuit can then be simplified to the one shown in Figure 8, where $Z[\leftarrow]$ and $Z[\rightarrow]$ are the resulting impedances of all previous T-sections to the left and right of the ceramic sections, respectively.

As is evident from Figure 8, the mechanical impedance presented to the transformer can be written as:

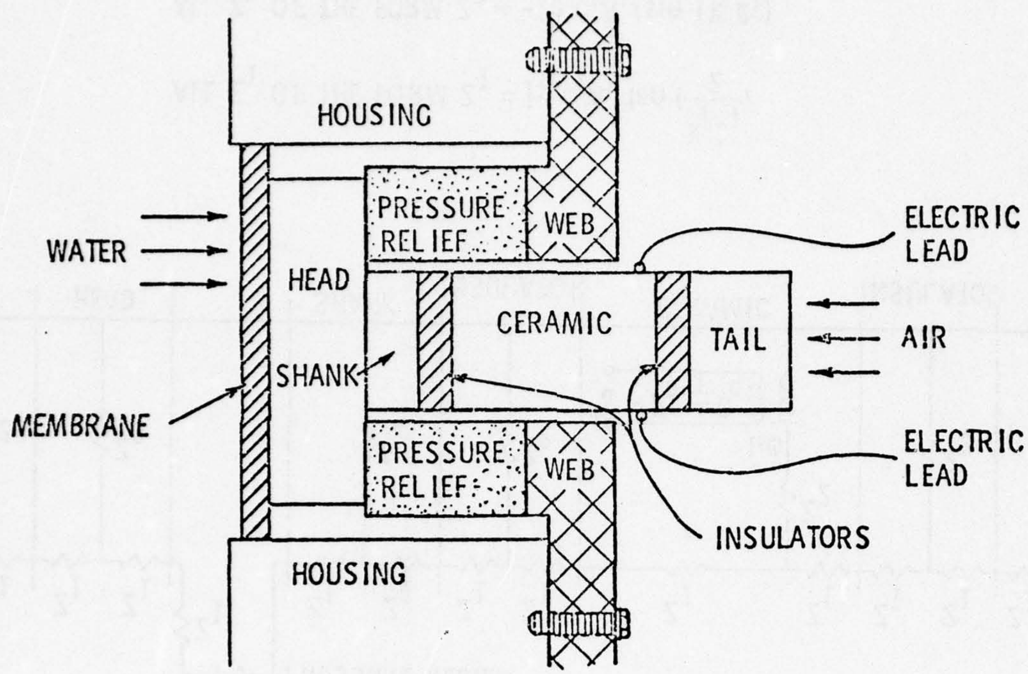
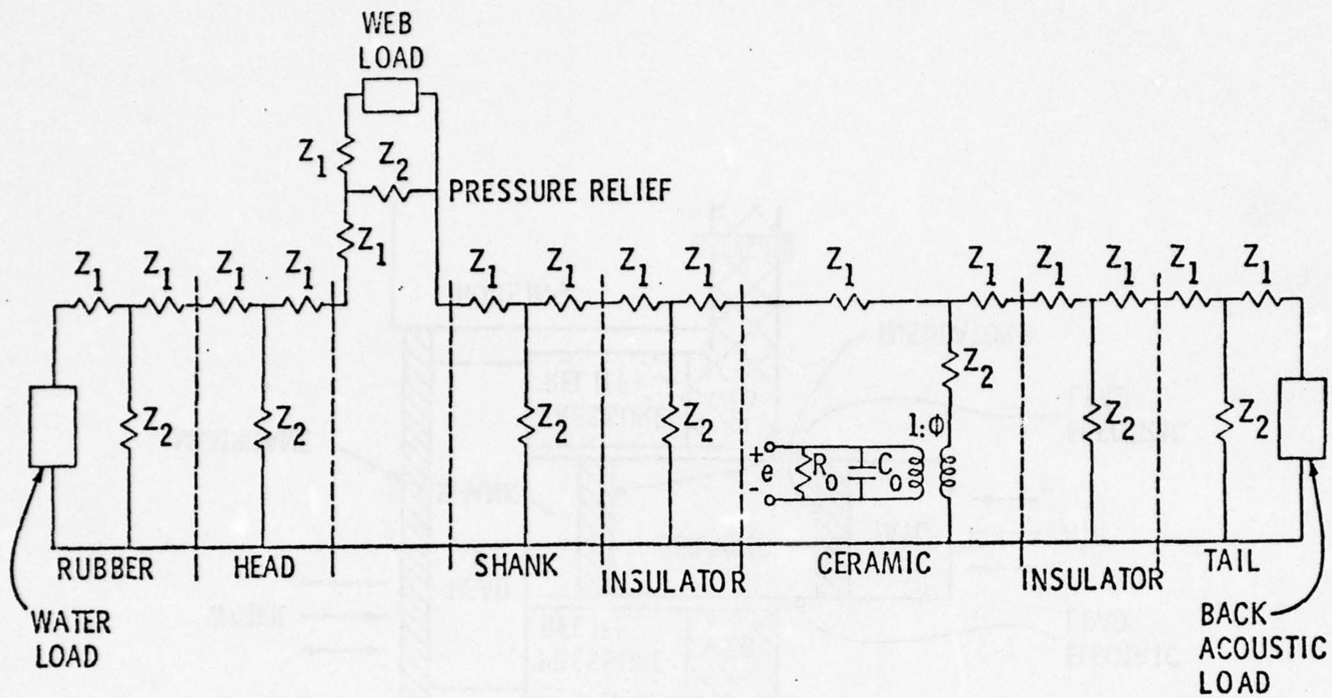


Figure 6. Transducer Mounted in Its Housing.



ALL Z_1 OF THE FORM $Z_1 = j\rho_i c_i A_i \tan\left(\frac{k_i \ell_i}{2}\right)$

ALL Z_2 OF THE FORM $Z_2 = -j\rho_i c_i A_i / \sin(k_i \ell_i)$

Figure 7. Circuit Representation of Transducer in Housing.

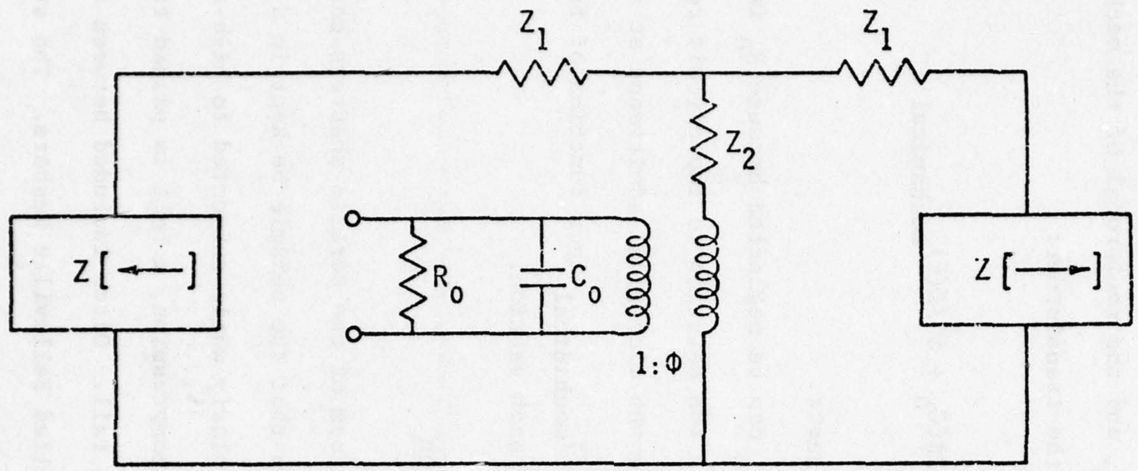


Figure 8. Reduced Transducer Circuit. The elements labeled Z_1 and Z_2 refer to the ceramic, and Z[←] and Z[→] represent left and right loads.

$$Z(f)_{\text{mechanical}} = Z_2 + [Z_1 + Z[\leftarrow]]\ [Z_1 + Z[\rightarrow]]/(Z[\leftarrow] + Z[\rightarrow] + 2Z_1)\ . \tag{9}$$

The input electrical admittance $\mathcal{Y}$ is the sum of the electrical admittances, $1/R_0$ and $j2\pi fC_0$, and the reciprocal of the mechanical impedance reflected through the transformer:

$$\mathcal{Y} = 1/R_0 + j2\pi fC_0 + \phi^2/Z(f)_{\text{mechanical}}\ , \tag{10}$$

where f is the frequency in hertz.

The term containing $1/R_0$ can be neglected because R_0 is normally very large. Equation (10) is the solution to the circuit representation and is used to calculate the electrical admittance at any frequency. The impedance $Z(f)_{\text{mechanical}}$ is a function of frequency through the wave number k of each section.

2.3 Stress-Bolt Consideration

The glassine molecular form of the ceramic shatters easily under tension, making it imperative that the ceramic be kept in a state of constant compression, particularly while subjected to high-electric fields. To accomplish this compression, a bolt is passed through the ceramic center from shank to tail. Often included between the tail and the bolt head are springs called Belleville washers. The stress rod (neglecting Belleville washers) can be modeled in Mason's Equivalent Circuit as another T-section joined to the shank and the back of the tail [7]. Figure 9 represents a drawing of this circuit.

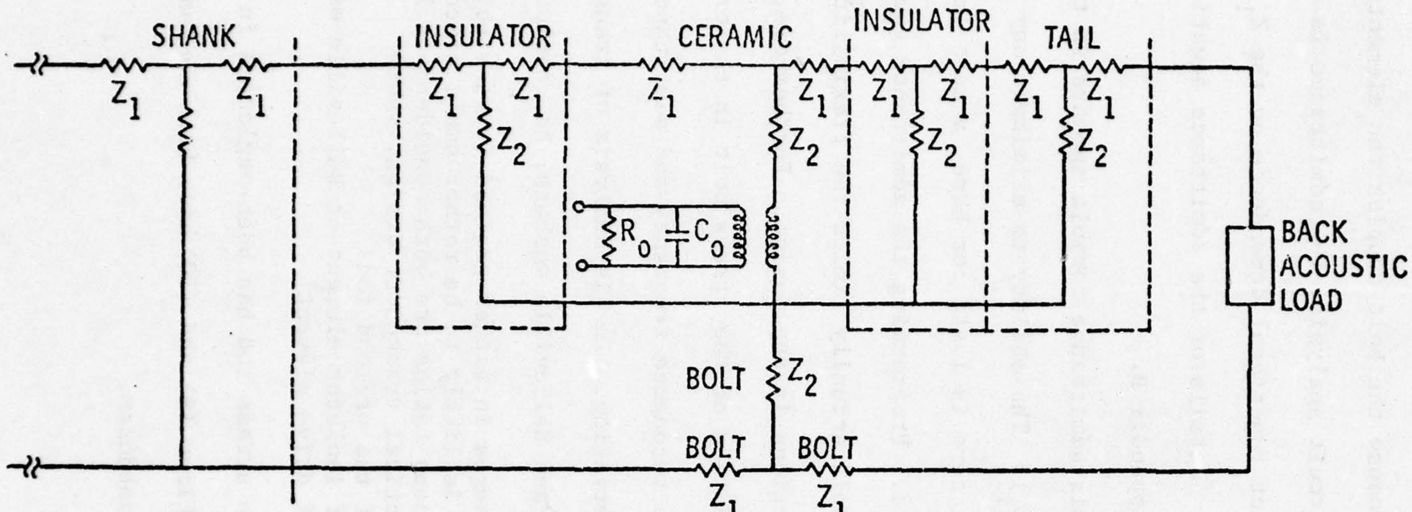


Figure 9. Transducer Circuit with Stress Bolt.

Evaluation of the circuit indicated in Figure 9 is difficult using the cascade Equation (8) because the bolt couples the elements together. Falling back on standard circuit analysis, the admittance is determined and is found to have a direct functional dependence on the Z_1 and Z_2 elements in each T-section. Details of the admittance equation of this circuit are given in Appendix B.

The difficulty with this admittance formula appears in the terms involving $Z_2 = -jZ_i/\sin(k_i \ell_i)$. The ability to eliminate any section by letting its length equal zero is lost; for here, a zero length generates a division by zero. Programming the admittance equation which includes the stress bolt actually limits the flexibility of choosing or eliminating sections in the design. Furthermore, practice has shown that the addition of the stress bolt in the transducer has a minimal effect on the resonance frequency and admittance. Brickman confirms this observation. In his analysis of transducers, preloaded with stress rods and Belleville washers, he concludes [10]:

> The effect of changes in either preload (stress bolt) or isolator stiffness is likely to be rather small since the K_p/K_q and K_r/K_q stiffness ratios are both considerably less than unity in any practical transducer design, where
> K_p = stiffness of the preload rod
> K_r = stiffness of isolator element or Belleville washer
> K_q = stiffness of drive element

For these reasons, the stress rod has been neglected in the transducer model, and Equations (9) and (10) have been chosen to mathematically represent the transducer.

2.4 The Admittance Loop

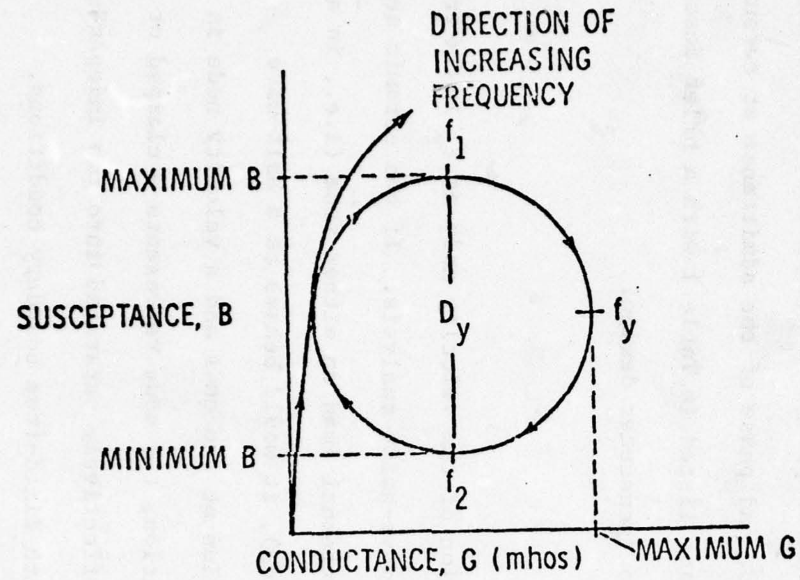
When Equation (10) is plotted in the complex plane with frequency as a parameter, the admittance loop shown in Figure 10 is obtained.

Five important design parameters can be calculated from the information contained in the admittance loop. They are the quality factor, the electromechanical coupling coefficient, the first resonance frequency, and the magnitude and phase of the admittance at resonance [5, 7]. These quantities are listed in Table 1 with a brief description of their importance to transducer design.

2.5 The Node Position

The approximate position of the velocity node at f_y in the transducer can be determined using a wave-guide analysis. If the ceramic section were operated without an external load on either face (i.e., in a free-free boundary condition), it would behave as a half-wave resonator with maximum motion at the ends and a velocity node in the center. In this configuration, the node represents a clamped or fixed point and the ceramic is effectively separated into two independent quarter-wave resonators with fixed-free boundary conditions.

If the other sections are added to the ceramic, the boundary conditions on these quarter-wave resonators change from fixed-free to fixed-loaded. To derive the equations to predict the nodal position, the transducer must be represented by a wave guide with interfaces between the loads and the ceramic at A and B, as shown in Figure 11.



f_y = THE ADMITTANCE RESONANCE FREQUENCY OR THE FREQUENCY OF MAXIMUM CONDUCTANCE G

f_1 = THE FREQUENCY OF MAXIMUM SUSCEPTANCE B

f_2 = THE FREQUENCY OF MINIMUM SUSCEPTANCE B

$\left|\mathcal{Y}(f_1) - \mathcal{Y}(f_2)\right|$ = THE DIAMETER OF THE LOOP, ALSO CALLED D_y

Figure 10. Admittance Loop of Transducer Circuit with Important Frequencies Marked.

Table 1

Five Important Design Parameters

Attribute	Relation to Points on Loop	Importance to Design
Mechanical quality factor[a] Q_m	$Q_m = \frac{f_y}{(f_2 - f_1)}$	A measure of the sharpness of the resonance peak
Resonance frequency f_y	Frequency of maximum conductance	Critically related to the sensitivity and magnitude of the output
Magnitude of admittance $\lvert y \rvert$	$\lvert y \rvert = \sqrt{G^2 + B^2}\,\Big\rvert_{f_y}$	Important for electronics matching
Phase of the admittance at resonance ψ	$\psi = \tan^{-1}(B/G)\,\Big\rvert_{f_y}$	Important for electronics matching

Table 1 (Continued)

Attribute	Relation to Points on Loop	Importance to Design
Effective electromechanical coupling coefficient k_{eff}	$k_{eff} = \frac{Cmot}{Cmot + CC}$ where, $CC = \frac{Imag(\mathcal{Y}(f_y))}{2\pi f_y}$ and[b] $Cmot = \frac{D_y}{2\pi f_y \cdot Q_m}$	Can be related to the sensitivity and to the bandwidth

[a]The quality factor Q_m should be represented by the resonance frequency divided by the difference of the half-power or half-maximum conductance points. However, because these latter two points are rather difficult to obtain with computer techniques, and because the admittance loop is essentially a circle and $1/R_0$ is very small, the half-conductance points are approximately equal to the maximum and minimum susceptance points.

[b]The value of the motional capacitance is dependent upon the diameter of the admittance loop as measured parallel to the real axis; however, by assuming a circular loop, the diameter measured parallel to the imaginary axis was used as an approximation to the correct diameter.

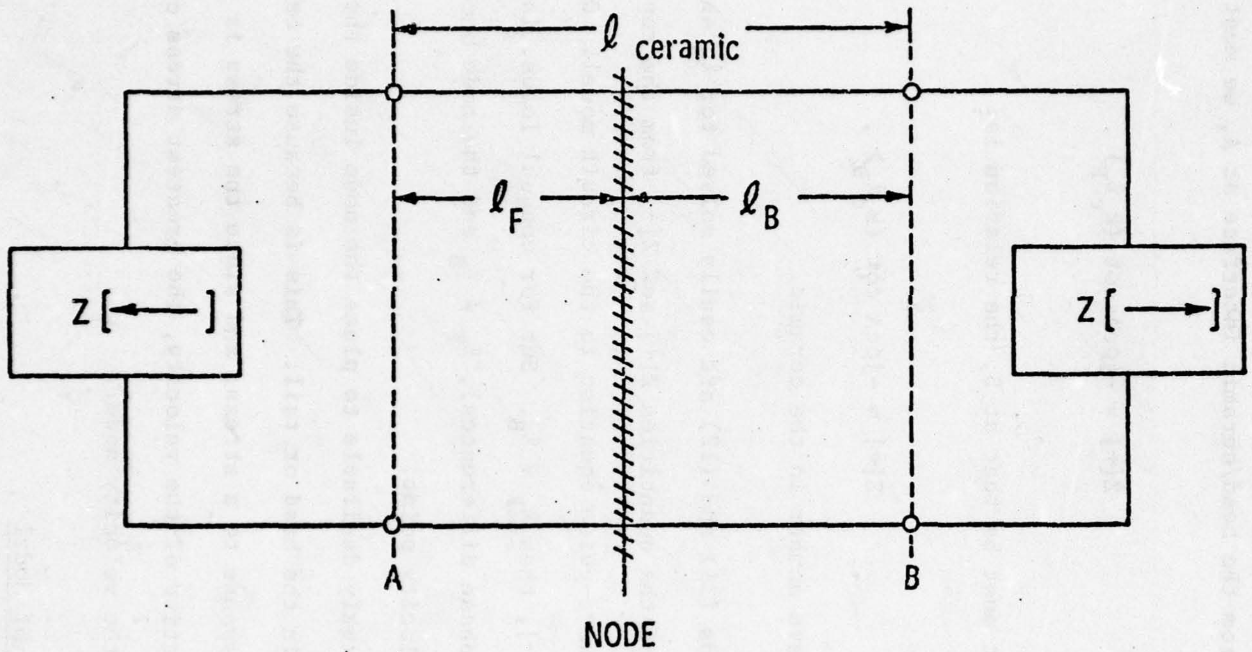


Figure 11. Wave-Guide Model of Ceramic with External Loading. The dimension ℓ_F equals the length forward from the node and ℓ_B is the length behind the node.

The condition of continuity of force and velocity demands equality of mechanical impedances at the interfaces. The impedance looking into the ceramic from A toward the node is $-j\rho cA \cot(k_c \ell_F)$. Therefore, from the head/ceramic interface at A, we must have:

$$Z[\leftarrow] = -j\rho cA \cot(k_c \ell_F) \ . \tag{11}$$

Similarly, it must be that at B, the relation is:

$$Z[\rightarrow] = -j\rho cA \cot(k_c \ell_B) \ , \tag{12}$$

where k_c = wave number in the ceramic.

Equations (11) and (12) are easily solved for ℓ_B and ℓ_F after calculation of the quantities $Z[\leftarrow]$ and $Z[\rightarrow]$ from the repeated application of the wave-guide equation to the circuit model. Of course, if $Z[\leftarrow] = Z[\rightarrow]$, then $\ell_F = \ell_B$. But for unequal loads, including real components (phase differences), $\ell_F \neq \ell_B$ and the node becomes merely a minimum velocity point.

It is highly desirable to place the node inside the ceramic rather than in the head or tail. This is because the ceramic develops an EMF in response to a stress; and since the stress is proportional to the derivative of the velocity, the greatest stress occurs in the vicinity of the velocity node.

2.6 Summary of Model

The transducer and its associated housing are represented by T-sections cascaded onto Mason's Equivalent Circuit (Figure 7).

The circuit is solved for its electrical input admittance by means of Equation (8). This admittance is repeatedly evaluated for increasing frequency; and as the admittance loop is traced out, its frequencies of interest are selected. Table 1 presents the attributes of the transducer which are evaluated from these frequencies and concomitant admittances. The position of the velocity node is determined by the solution of the circuit in the same manner inward to the ceramic and application of Equations (11) and (12).

2.7 A Verification of the Computer Algorithm

In order to assess the validity of the computer algorithm used to predict the correct resonance frequency, a three-section transducer was postulated as a test case. Sections one and three were composed of brass, while section two was piezoceramic. All sections had equal cross-sectional areas and were of equal lengths. Such a simplification could represent an unmounted transducer in vacuum. To test the algorithm for frequency, the parameters of the hypothetical three-section transducer as shown in Table 2 were inserted into the input. The other available sections of the design model were removed by letting their lengths equal zero. A small loss factor of 0.001 was permitted in the ceramic, and the web and radiation loads were set to zero corresponding to an unmounted transducer with air (vacuum) loading. The magnitudes of the clamped capacitance, C_0, and the transformer turns ratio ϕ were arbitrary, as these do not affect the value of the frequency at the fundamental resonance. Using these data, the computer

Table 2

Input Parameters for the Model of Figure 7[a]

Transducer Model	Test Case	Density (ρ)	Bar Velocity (c)	Length (ℓ)	Area (A)	Loss Factor (η)
Head	Section 1	1.741×10^3	4800	.0390	$.238 \times 10^{-3}$	0
Shank	NA	1	1	0	1	0
Insulator	NA	1	1	0	1	0
Ceramic	Section 2	7.501×10^3	2919	.0390	$.238 \times 10^{-3}$	.001
Insulator	NA	1	1	0	1	0
Tail	Section 3	1.741×10^3	4800	.0390	$.238 \times 10^{-3}$	0
Pressure Relief	NA	1	1	0	1	0
Rubber Window	NA	1	1	0	1	0

[a] These data apply to the three-section transducer with brass head and tail, and small losses. All units are SI units.

algorithm evaluated the admittance and selected the frequency of maximum conductance, the resonance frequency, as $f_y = 21770$ Hz.

It can also be shown that the resonance frequency can be calculated by demanding equality of impedance at any interface. This leads to the development of a relation for any three-section composite half-wavelength longitudinal vibrator:

$$\tan(k_2\ell_2) = \frac{B_1 \tan(k_1\ell_1) + B_3 \tan(k_3\ell_3)}{B_1 B_3 \tan(k_1\ell_1) \tan(k_3\ell_3) - 1}, \tag{13}$$

where

$$B_1 = \frac{\rho_1 c_1 A_1}{\rho_2 c_2 A_2}$$

and

$$B_3 = \frac{\rho_3 c_3 A_3}{\rho_2 c_2 A_2},$$

with A = area, ρ = density, c = bar velocity, k = wave number, and ℓ = length.

The Secant Method was chosen to find the roots to Equation (13). From the data shown in Table 2, the resonance frequency of the three-section test case given by this method was 21769.221 Hz. This is in excellent agreement with the value obtained from the computer algorithm and so lends confidence to the use of this routine.

As is well known [9], the presence of losses in a parallel circuit decreases the resonance frequency. The ceramic transducer is in essence a parallel circuit as shown in Figure 8 and, therefore, should suffer a decrease in its resonance frequency for increased losses. To display this, the data from Table 2 were rerun with the

loss factor of the ceramic section set equal to 0.2 instead of the previous value of 0.001. The result is a smaller resonance frequency as expected:

$$f_y = 21560 \text{ Hz} .$$

2.8 A Verification of the Model

Five quite different transducers constructed with a variety of materials were selected to serve as inputs for the algorithm which evaluates the transducer model. Physical dimensions of each transducer were measured, and tests using the Vector Admittance Locus Plotter (VALP) were made on each to determine the frequencies of resonance. Quality factors Q_m were obtained from f_1 and f_2, being the frequencies of maximum and minimum susceptance, respectively, and from the resonance frequency f_y by:

$$Q_m = \frac{f_y}{f_2 - f_1} . \tag{14}$$

The effective coupling coefficient k_{eff} was estimated from the approximate equation:

$$k_{eff} = \sqrt{1 - \frac{f_y^2}{f_z^2}} , \tag{15}$$

where f_z is the approximate frequency of minimum conductance.

Values of the magnitude of the admittance at resonance were recorded. In two cases, the position of the node was measured by placing an accelerometer probe on the ceramic and searching for the position of minimum longitudinal movement.

Table 3 shows, for the five differently constructed transducers, the percentage differences between the values returned by the program and those measured experimentally.

On averaging the first four values in column 1, it can be seen that the program's calculated frequencies are approximately 7.1% higher than the mean measured resonance frequencies. In order to see whether this discrepancy could be ascribed to such neglected factors as the finite thickness of the electrodes and glue joints, additional lengths of 1.0 mm were added in the program to each of the insulators. However, this adjustment only lowered the calculated resonance frequencies by an average of 0.3%. Therefore, it is thought that other mechanisms such as transverse vibrations, head flap, or compliance of the glue must account for this shift. The disparate value of a 14.4% difference for transducer #5 was assumed to be caused by a defect in this transducer. It is not uncommon for a transducer to have a poorly glued joint which manifests itself as a widely deviated resonance frequency, or even quite often as two resonances. In fact, one of the tests for this condition is to compare the resonance frequency with a sampling of others. Insofar as this aspect is concerned, transducer #5 may presumably be discounted.

Column 2 contains the percentage differences between calculated and measured quality factors. Normally, with air loading on both

Table 3

Percentage Differences Between the Program-Generated

Design Values and the Measured Values

Transducer #	$\%\Delta f_y$	$\%\Delta Q_m$	$\%\Delta k_{eff}$	$\%\Delta\lvert Admit\rvert(m\mho)$	$\%\Delta \ell_F$
1	7.20	- 6.47	-57.00	60.30	-40.05
2	6.68	31.60	23.20	-58.10	- 6.10
3	8.71	9.47	- 3.53	60.50	NA
4	5.98	-11.07	- 8.03	22.70	NA
5	14.40	40.10	-56.00	75.00	NA

acoustic ports, a transducer's Q_m will lie between 200 and 500. The differences in column 2 are well within the deviations encountered in a set of "identical" elements. The quality factor is a parameter which is greatly dependent on the real part of the external load, as well as on the internal losses of the system. When the input data for the five transducers were rerun using water loading on the faces, Q_m was seen to fall to values ranging between 5 and 9, while at the same time, the frequency f_y shifted very slightly downward. This behavior is confirmed by experiment [9].

The coupling coefficient discrepancies are displayed in column 3. Here, there is very poor agreement between calculations and measurement. It is felt that the measured k_{eff} is more probably in error because of the approximations involved in the measurement technique using the VALP. The calculated values for k_{eff} in air range from 0.50 to 0.65 and are probably more reliable. In addition, for water loading, the calculated k_{eff} increased on average by 14% in agreement with experimental observations [9].

Column 4 deals with the absolute value of the admittance at resonance. These percentage differences are well within the variability shown by a group of "identical" transducers, and agreement is considered to be good. For water loading, the program-generated values for the magnitude of the admittance in all cases decreased by two orders of magnitude, as expected.

The last column of Table 3 contains results for the nodal position in two of the transducers. Agreement was reasonable for transducer #2, but rather poor for #1. Generally, it is expected that the

calculated position of the node will be accurate enough for use in the design program.

CHAPTER 3

GOAL PROGRAMMING

3.1 General Theory

In the Nonlinear Goal Programming formulation (NLGP) chosen for this design problem, the independent variables are assigned to the vector of unknowns $\bar{x}$. The dependent variables are called the objective functions, and are the components of the vector $\bar{Y}$. The initial step in the goal-programming formulation is the heuristic development of a set of mathematical statements, which are functions of the independent variables $\bar{x}$ to describe the various design objective functions.

The general form of the equation relating objective function to desired value is:

$$Y_i(\bar{x}) + n_i - p_i = b_i \ . \qquad (16)$$

Here, the vector element b_i is the desired achievement or optimum value acceptable for Y_i. In some cases, the number b_i may actually be unattainable, whereupon a reasonable goal must be established. For example, it may be desirous to have a cost objective equal to zero; however, a more reasonable goal of 20% of the estimated cost should be assumed.

The other two variables introduced in Equation (16) are the design deviation variables, n_i and p_i. These nonnegative variables are,

respectively, the amount by which Y_i is under or over its goal, b_i. To achieve the desired right-hand side, certain of the deviation variables must be driven to zero. To strive for an exact fulfillment of a goal, both the negative and positive deviations will be minimized, whereas if the goal is unattainable only one of the deviation variables will be necessary.

All of the pertinent n_i and p_i are multiplied by weights w_i and summed in priority levels to create the achievement function $\bar{a}$, as defined in Equation (17):

$$\bar{a} = \left[\sum_k (w_k^+ n_k + w_k^- p_k) \; , \; \sum_j (w_j^+ n_j + w_j^- p_j), \; . \; . \; . \right] , \tag{17}$$

where k = index of priority level 1.

j = index of priority level 2.

The achievement function is split, vector fashion, into priority levels. The deviation variables relating to the primary objectives, those which <u>must</u> be satisfied, are summed in priority level #1. Minimization in this priority level is tested before consideration of the next lower level, and this level's solution can never be degraded. The remaining n_i and p_i from the secondary functions are contained in other lower priority levels. A lexicographic minimum is sought for the achievement function. Lexicographic minimum implies an ordered minimum in which the priority levels are considered sequentially. For example, by this designation, the vector (2, 20000) is a smaller vector than (5, 1).

Also included in the achievement function are weight factors. These weight factors $\bar{w}$ can be extremely important in a problem with

diverse objectives. Since the magnitude of n_i or p_i may differ by several orders of magnitude for different objectives at the same priority level, it is necessary to reflect their relative importance in the achievement function. Accordingly, a weight factor is assigned to each n_i and p_i within each priority level. These weight factors also provide versatility to change the importance of one or another term within the achievement function.

The entire NLGP technique for optimization using two priority levels can be succinctly expressed as follows [1]:

Locate the transducer design variables $\bar{x}$ so as to lexicographically minimize $\bar{a}$:

$$\bar{a} = \left\{ \sum_k (w_k^+ n_k + w_k^- p_k),\ \sum_j (w_j^+ n_j + w_j^- p_j) \right\},$$

such that:

$$Y_i(\bar{x}) + n_i - p_i = b_i\ , \qquad \text{for all } i = 1 \ldots m$$

where: k = summation index for primary objectives in priority level #1,

j = summation index for secondary objectives in priority level #2,

m = # of objective functions (m = k + j).

The method of pattern search begins by changing a variable by a small amount, Δ, and evaluating the resultant achievement function. If the new achievement function is smaller by a specified tolerance than the old one, the Δ step is deemed successful. The variable is then promoted temporarily to the new value, and the program progresses to test the next variable. At the end of a successful search, all variables are accelerated (each in the direction of its successful Δ) by an acceleration factor α. The achievement function is perturbed

at this accelerated point, and if the result is smaller than before, the process begins anew. If no improvement is found, the search falls back to the previous base point, the size of each Δ is reduced, and the search is begun again.

There are three methods for reaching convergence to an optimum compromise. The first, the most obvious, is to completely satisfy all goals and reach $\bar{a} = (0, 0)$. Failing that, the program looks at the size of the change in $\bar{a}$ after each perturbation. If the change is less than 0.001 in each priority level, then the program assumes a local optimum point has been reached and declares success. If neither of these states occur, the program stops with a best-vector-to-date for $\bar{x}$, after using the maximum number of cycles or reductions the operator has specified.

The NLGP code chosen for the transducer design program is an early, simple version of Nonlinear Goal Programming. More powerful codes exist which can solve problems with 80,000 variables and several thousand objective functions and at the same time are (typically) many orders of magnitude faster while requiring less storage [11]. However, for this small problem, the simplified version is adequate.

3.2 Application to Transducer Design

Applying the NLGP technique to transducer design, the sections of the transducer and housing are numbered according to Figure 12. The possible independent variables $\bar{x}$ are (for each of the eight transducer sections) length, cross-sectional area, density, bar velocity, and loss factor, for a total of 40 possible variables.

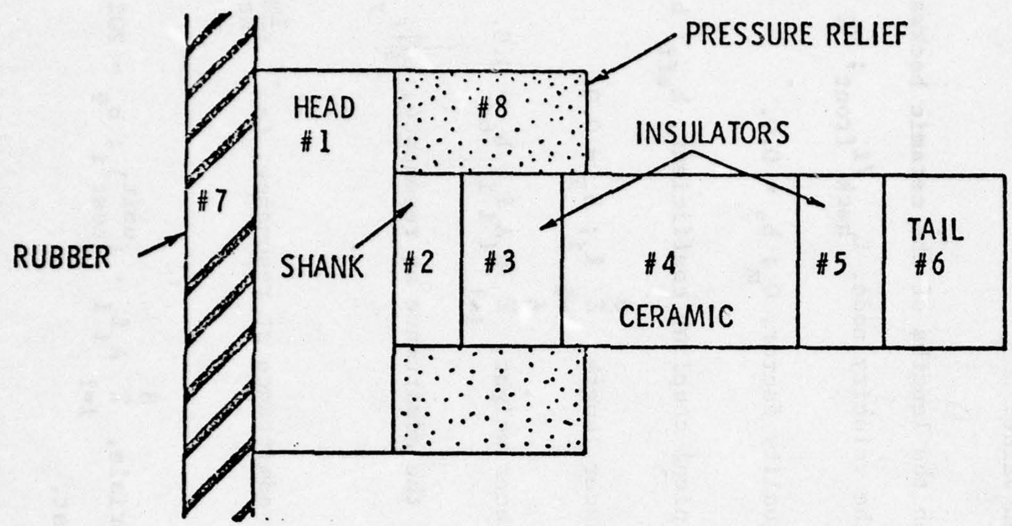


Figure 12. Transducer Section Numbers Used in NLGP Program for Design.

Currently, 11 objective functions are defined. These are not the only objectives which might be chosen, and others can easily be added or substituted. The goals b_i are chosen as typical transducer specifications. These objective functions are:

$Y_1(\bar{x})$ = primary admittance resonance frequency, f_y, and b_1 = specification value.

$Y_2(\bar{x})$ = ratio between the lengths of the ceramic backward and forward of the velocity node, ℓ_{back}/ℓ_{front}; $b_2 = 1.0$.

$Y_3(\bar{x})$ = mechanical quality factor, Q_m; $b_3 = 0.0$.

$Y_4(\bar{x})$ = electromechanical coupling coefficient, k_{eff}; $b_4 = 1.0$.

$Y_5(\bar{x})$ = total transducer length, $\sum_{i=1}^{6} \ell_i$; $b_5 = 0.0$.

$Y_6(\bar{x})$ = total transducer weight, $\sum_{i=1}^{6} \rho_i A_i \ell_i$; $b_6 = 0.0$.

$Y_7(\bar{x})$ = magnitude of the admittance at resonance, $|\mathcal{Y}|_{f_y}$; b_7 = 0.1.

$Y_8(\bar{x})$ = phase of the admittance at resonance, $\tan^{-1}\left(\frac{\text{Imag}\,\mathcal{Y}}{\text{Real}\,\mathcal{Y}}\right)_{f_y}$; $b_8 = 0.0$.

$Y_9(\bar{x})$ = cost of materials, $\sum_{i=1}^{8} A_i \ell_i \cdot \left(\frac{\text{unit}}{\text{cost}}\right)_i$; b_9 = 20% of estimated cost.

$Y_{10}(\bar{x})$ = ratio of the length behind the node to the total length of ceramic, $\ell_{back}/\ell_{ceramic}$; $b_{10} = 1.0$.

$Y_{11}(\bar{x})$ = ratio of the length of the ceramic in front of the node to the total length of ceramic, $\ell_{front}/\ell_{ceramic}$; b_{11} = 1.0.

These objective functions are evaluated in the design program in the subroutine YVALUE following the methods and equations given in Section 2.6.

For most typical "Tonpilz" design problems, the achievement function is written as:

$$\bar{a} = \Big[(w_{10}^- p_{10} + w_{11}^- p_{11}),(w_1^+ n_1 + w_1^- p_1 + w_2^+ n_2 + w_2^- p_2 + w_3^- p_3 + w_4^+ n_4 + w_5^- p_5 + w_6^- p_6 + w_7^+ n_7 + w_8^- p_8 + w_9^- p_9)\Big] . \qquad (18)$$

This formulation shows the placement of the deviation for objective functions 10 and 11 in priority level #1. Objective function $Y_{10}(\bar{x})$ is the ratio $\ell_f/\ell_{ceramic}$, and objective function $Y_{11}(\bar{x})$ is $\ell_b/\ell_{ceramic}$. To insure the node remains inside the ceramic, neither of these ratios should become greater than unity. Therefore, the b_i of these objective functions are set equal to unity. There is a definite advantage to placing the deviations of these ratios in priority level one. If the starting point or initial guess for the variables is chosen so as to place the node inside the ceramic, then the nodal ratios are less than unity, and p_{10} and p_{11} will both be zero after the first achievement function test. As the variables are perturbed in the search for a lexicographic minimum, this priority level solution of zero cannot be degraded, so that now, all future solutions are guaranteed to have their node inside the ceramic boundaries.

Table 4 displays a list of some of the important variables of the NLGP code for transducer design. Table 5 provides a User's Guide to the input for this program. The entire program is listed in Appendix C.

3.3 Variables

Any of the parameters such as density ρ, bar velocity c, area A, length ℓ, or loss factor η of each section may be chosen as a variable to be optimized by the NLGP technique. For proper operation, the selected variables must be placed as components in the vector $\bar{x}$. This takes place in the subroutine "VARIB" at the end of the program deck. For example, to let the density of section 1 and the sound velocity of section 3 be variables, cards containing the following assignments must be placed in "VARIB."

```
RHO(1) = X(1)
C(3) = X(2)
```

For theoretical research, any of the material parameters may be variable; however, for practical design, there are some limitations. In actual practice, the product of density and bar velocity, the specific acoustic impedance, is the quantity of interest, and materials exhibiting the necessary combination are limited by availability and cost to two or three (in each section). It is far easier to fix the specific acoustic impedances throughout and then to optimize the remaining parameters than to try to obtain the materials corresponding to a program-generated solution.

Table 4

Definition of NLGP Code Variables

Variables	Utility
NVAR	Number of Decision Variables
NOBJ	Number of Objective Functions
NPRIOR	Number of Priority Levels
NACH	Number of Terms in Achievement Function
NMAX	Maximum Number of Pattern Search Cycles Allowed
X	Vector of Decision Variables at Test Point
EPS	Vector of Step Sizes Used in Pattern Search
RHS	Right-Hand-Side Values of Objective Functions
MPRI	Priority Level of Given Deviation Variable
WEIGHT	Weight Factor of Given Deviation Variable
ALPHA	Pattern Search Acceleration Factor
BETA	Step Reduction Factor
MAXRED	Maximum Number of Step Reductions Allowed
IPRINT	Output Switch--0 Gives Only Final Result; 1 Gives Result of Every Pattern Search Cycle
EPSY	Vector of Improvement Tolerances in Achievement
SIGN	Indicates Positive or Negative Deviation to be Minimized from Objective Function IROW at MPRI Level with Weight Factor Weight
IROW	Objective Function of Given Deviation Variable
Y	Vector of Functions of Decision Variables
XBASE	Base Point for Hooke-Jeeves Algorithm
ZTEST	Achievement Function Value Vector at a Test Point
RHO	Vector of Density Values (MKS) KG/CU M
C	Vector of Sound Speeds, m/sec
L	Length of Each Wave-Guide Section, M
A	Area of Each Section Square Meters
ETA	Vector of Loss Factors
TURNS	The Square of the Transformation Factor
CAP	The Calculated Capacitance
ZW	The Loading Presented to the Front Face
ZPR	The Loading Presented by the Web Applied to the Pressure Relief
STEP	The Frequency Increments Used to Find Resonance by Iteration
COST	Vector of Values of Each Cost Per Unit Volume

Table 5

A User's Guide to the Input

Variable	Format
Card 1 NVAR, NOBJ, NPRIOR, NACH, NMAX	5I5
Card 2 IPRINT, MAXRED	2I5
Card(s) 3- X(I), I=1, NVAR	8E10.4
Card(s) 4- EPS(I), I=1, NVAR	8E10.4
Card(s) 5- RHS(I), I=1, NOBJ	8E10.4
Card 6 ALPHA, BETA	2E10.4
Card 7 EPSY(I), I=1, NPRIOR	8E10.4
Cards 8-22 SIGN(I), IROW(I), MPRI(I), WEIGHT(I), I=1, NACH	F5.0, 2I5, F10.0
Cards 23-24 COST(I), I=1, 8	8E10.4
Cards 25-34 RHO(I), C(I), L(I), A(I), ETA(I), I=1, 8	8E10.4
Card 35 TURNS, CAP, ZW, ZPR NOTE: ZW AND ZPR ARE COMPLEX AND REQUIRE 2 DATA FIELDS EACH	6E10.4
Card 36 STEP	I5

While at first glance it would seem advantageous to include the cross-sectional area as a variable, there are strong objections to this. The area of the head is usually fixed by array dimension and wavelength requirements. The outside diameters of the shank, insulators, and tail are usually chosen to conform to the outside diameter of the ceramic for purposes of alignment. In turn, the ceramic can be purchased most inexpensively if it is selected in stock sizes. Finally, a most important consideration is the evaluation of the capacitance, C_0, which strongly affects the value of the coupling coefficient and also the value of the admittance at resonance. The area of the ceramic presented to the plane wave is not necessarily the area of the electrodes which define the capacitance. There are many electrode configurations other than the sandwich type, such as cylindrical shells with the electrodes inside and outside, or in stripes down the sides. Thus, if the area of the ceramic face is to be a variable, the capacitance must be made to assume variability, also, in the appropriate manner. No attempt has been made to program all the variety of capacitance relations possibly applicable. Instead, both area and length of the ceramic should be fixed, and the capacitance calculated as required. It is felt that this method will provide the most flexibility in geometry and electrode position for transducer design.

Finally, for practical purposes, the loss factor should probably be kept constant because it might be extremely difficult to find materials which match all the other desired properties as well as any specification on losses.

3.4 Weight Factors in the Achievement Function

The weight factors w_i (multipliers of the deviation variables n_i and p_i) are extremely important to proper program operation. In order to have an order of magnitude equality among all the terms of the achievement function, it is necessary to examine the anticipated size of the deviations. The goals of the example in Section 4.1 are examined in Table 6 to illustrate the assignment of weight factors. Regarding the assignments of the w_i shown, there are two points which should be made. First, all of the deviations are slowly changing with the exception of the one associated with the resonance frequency. The weights assigned for this deviation will emphasize the solution for f_y until this frequency approaches its goals, whereupon the other objectives begin to assume importance. Secondly, the second objective function, ℓ_{back}/ℓ_{front}, is to be optimized toward 1.0, and failing this, it is preferred that the ratio be smaller than unity so that the node will be more toward the tail. The following argument demonstrates the direction of the emphasis of the solution. When the fraction is less than 1.0, p_2 is zero and n_2 can only take values less than 1.0. If the ratio is greater than 1.0, n_2 is zero but p_2 can assume any value. Thus, there will be a larger influence on the summation $\bar{a}$ if the deviation is positive, and this means the solution will be more concerned with lowering p_2 than raising n_2, a desirable result. The rest of the weights are assigned in a straightforward manner according to the anticipated size of the respective deviations.

Table 6

Order of Magnitude Estimation Used to Assign Weight Factors[a]

$Y_i(\bar{x})$ Objective	b_i Goal	Expected Deviation	w_i Weight
1. resonance frequency[b] f_y	5000	$0 \leq n_1 \simeq 500$ or	0.1
		$0 \leq p_1 \simeq 500$	0.1
2. ℓ_{back}/ℓ_{front}	1	$0 \leq n_2 \leq 1$ or	10
		$0 \leq p_2 \leq \infty$	10
3. Q_m	0	$p_3 \approx 10$ (water)[c]	1
4. k_{eff}[c]	1	$n_4 \approx .5$	10
5. length[b]	0	$p_5 \approx .3$	10
6. weight[b]	0	$p_6 \approx 6$	1
7. |admittance|[c]	0.01	$n_7 \approx .009$	1000

Table 6 (Continued)

$Y_i(\bar{x})$ Objective	b_i Goal	Expected Deviation	w_i Weight
8. phase[c]	0	$0 \leq p_8 \simeq .2$	100
9. cost[b]	20% of estimated cost	$p_9 \simeq$ 1-100	0 (for illustration)
10. $\ell_{back}/\ell_{ceramic}$	1	$p_{10} = 0$	1
11. $\ell_{front}/\ell_{ceramic}$	1	$p_{11} = 0$	1

[a] These apply to the example set in Section 4.1.

[b] Estimated from initial values.

[c] Average value.

CHAPTER 4

EXAMPLE OF DESIGN PROGRAM USAGE

4.1 An Illustrative Example

To display the inner working of the design program, the following example was chosen. Suppose one wished to calculate the lengths of the head, insulators, shank, and tail of a transducer necessary to provide an element resonant at 5000 Hz in water, with no housing, and with a combination of other design features as defined in Table 6. To begin the problem, decisions concerning the compositions of the sections and the length and area of the ceramic must be made. The capacitance and transformer turns ratio applicable to the geometry and material constants must be calculated (see Equations (4) and (5)). Initial lengths for the various sections to be optimized can be assigned by estimating the total transducer length. Assuming it to be a half-wavelength vibrator of ceramic, then:

$$\ell = \frac{c}{2f} , \tag{19}$$

where, f = the desired frequency of resonance.

c = bar velocity in the ceramic.

After the length of the ceramic (and other fixed-length sections) has been subtracted, the remaining length can be divided among the additional desired parts.

It should be emphasized that the purpose of this study is to develop and illustrate a design technique, not to provide specific transducer designs. For this example, the head and shank were chosen to be aluminum, the insulators were glass, and the tail was tungsten. Chosen for the ceramic were four discs, each of cross-sectional area equal to 2.38×10^{-3} square meters, and length 5.0×10^{-2} m, and having the material constants of Channelite #5400(R) [10]. The capacitance was calculated to be 1134 picofarads and the square of the turns ratio was 0.5576. The initial lengths of the head and tail were 5×10^{-2} m. One insulator length was 2.5×10^{-3} m while the other was 3.0×10^{-3} m. The shank length was chosen to be 3.0×10^{-3} m.

The assignment of the variables in the subroutine "VARIB" was made as shown in Table 7.

Table 8 contains a listing of all the input variables for this problem. Explanations of all variables can be found in Table 4. The radiation loading was set to the value of the specific acoustic impedance (ρc) of water. Since housing effects were to be neglected, the web load and the lengths of sections 7 and 8 were set equal to zero. Cross-sectional areas for the head and shank were arbitrarily selected while the other sections were the same in cross section as the ceramic.

The weight factors were assigned as given in Table 6. This is the case of "equal" weights, meaning that the terms in the achievement function are all approximately of the same order of magnitude. The step sizes EPS were chosen to be 1% of the initial value. Some consideration was given to the acceleration factor ALPHA. If the

Table 7

Variable Assignment in "VARIB" for the Problem Example

```
      SUBROUTINE VARIB
      REAL L(9)
      DIMENSION RHO(9), C(9), A(9), ETA(9), X(12)
      COMMON/COMM 12/RHO, C, L, A, ETA
      COMMON/COMM 02/X
C  User Supplied Variables Are Listed Here
      L(6) = X(1)
      L(5) = X(2)
      L(3) = X(3)
      L(2) = X(4)
      L(1) = X(5)
      RETURN
      END
```

Table 8

Program Input for Illustrative Example with "Equal" Weights

Card #	Description	0 (1↓ 5↓ 9↓) 10	10 (15↓ 19↓) 20	20 (24↓) 25 30	30 40	40 50	50 60	60 70	70 80
1.	NVAR, NOBJ, NPRIOR, NACH, NMAX	5 11	2 13	20					
2.	IPRINT, MAXRED	1 5							
3.	Variables--Initial Guess: ℓ_6, ℓ_5, ℓ_3, ℓ_2, ℓ_1	0.5000E-01	0.2500E-02	0.3000E-02	0.3000E-02	0.5000E-01			
4.	EPS--Step Sizes of Variables	0.001	0.0001	0.0001	0.0001	0.001			
5.	RHS--Goals b_i	5000	1.0	0.0	1.0	0.0	0.0	0.01	0.0
6.	RHS--Goals b_i	0.0	1.0	1.0					
7.	ALPHA, BETA	1.5	0.5						
8.	EPSY	0.001	0.001						

Table 8 (Continued)

Card #	Description	0 (1↓ 5↓ 9↓)	10 (15↓ 19↓)	20 (24↓)	30	40	50	60	70	80
9.	SIGN, IROW, MPRI,	1.0 10	1 1.00							
10.	WEIGHT--	1.0 11	1 1.00							
11.	defines achievement	-1.0 1	2 0.1							
12.	function	1.0 1	2 0.1							
13.	deviation variables	-1.0 2	2 10.0							
14.		1.0 2	2 10.0							
15.		1.0 3	2 1.0							
16.		-1.0 4	2 10.0							
17.		1.0 5	2 10.0							
18.		1.0 6	2 1.0							
19.		-1.0 7	2 1000.0							
20.		1.0 8	2 100.0							
21.		1.0 9	2 1.0							
22.	COST PER UNIT VOLUME	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
23.	RHO, C, L, A, ETA (1)	0.2701E04	0.5150E04	0.5000E-01	0.1000E-01	0.0				
24.	parameters of (2)	0.2701E04	0.5150E04	0.2500E-02	0.3550E-02	0.0				
25.	sections of (3)	0.3675E04	0.9360E04	0.3000E-02	0.2380E-02	0.0				
26.	transducers (4)	0.7501E04	0.2890E04	0.2000E00	0.2380E-02	0.4000E-02				
27.	(5)	0.3675E04	0.9360E04	0.3000E-02	0.2380E-02	0.0				
28.	(6)	0.7750E04	0.5100E04	0.5000E-01	0.2380E-02	0.0				
29.	(7)	1.0	1.0	0.0	1.0	0.0				
30.	(8)	1.0	1.0	0.0	1.0	0.0				

Table 8 (Continued)

Card #	Description	0 ↓1 ↓5 ↓9 10	↓15 ↓19 20	↓24 25 30	30 40	40 50	50 60	60 70	70 80
31.	TURNS, CAP, Z_W, Z_{PR} Transducer model constants and external loads	0.5576	0.1134E-08	0.1500E07	0.0	0.0	0.0		
32.	Frequency-loop step size	1							

acceleration is chosen to be too large, it causes the system to fluctuate back and forth about an optimal solution. In this condition, the program is said to "hunt to and fro" and the results are likely to be erratic. After some trial and error, a value of 1.5 was selected for ALPHA. The achievement tolerances EPSY were both set to 0.001.

The output for the case of "equal" weights is shown in Table 9. Each iteration is presented to show the orderly improvement in the objective functions. The output shows both the solution vector of lengths and the values of each objective function at those lengths. An examination of the sequence of outputs at each step reveals the resonance frequency climbing towards 5000 Hz, Q_m being reduced, and k_{eff} being increased, weight and length both being reduced, and the node approaching the center. To achieve this state, some of the variables are seen to vary in an interesting fashion about the center variable. The tail as well as all forward sections are reduced in length while the fifth section insulator is increased.

4.2 Variation of Weights

The next set of tables presents the outputs after the weight on one or another deviation variable was greatly magnified. This tends to steer the solution to favor optimizing that function over any other objective.

Regarding Table 10, the distorted importance of Q_m causes the objective function Y(3) to steadily decrease. To achieve this, the length of the head shrinks while the tail grows. Effectively, the

Table 9

Output for the Illustrative Example with "Equal" Weights

Objective Function	Objective Function at Starting Point	Position After 1st Search Pattern	Position After 2nd Search Pattern	Position After 3rd Search Pattern and Two Step Reductions
Y_1 (x) = resonance frequency	4515	4589	4740	5030
Y_2 (x) = ℓ_{back}/ℓ_{front}	1.31220	1.30580	1.29520	1.27600
Y_3 (x) = Q_m	7.36540	7.24960	7.03260	6.67100
Y_4 (x) = k_{eff}	0.60130	0.60240	0.60260	0.60140
Y_5 (x) = length	0.30850	0.30350	0.29370	0.27720
Y_6 (x) = weight	5.92010	5.80620	5.58770	5.21570
Y_7 (x) = abs (admittance)	0.00015	0.00015	0.00015	0.00015
Y_8 (x) = phase	0.22790	0.22990	0.23610	0.24860
Y_9 (x) = cost	0.00000	0.00000	0.00000	0.00000
Y_{10}(x) = $\ell_{back}/\ell_{ceramic}$	0.51400	0.51160	0.50710	0.49850
Y_{11}(x) = $\ell_{front}/\ell_{ceramic}$	0.39170	0.39180	0.39150	0.39000

Table 9 (Continued)

Variables	Objective Function at Starting Point	Position After 1st Search Pattern	Position After 2nd Search Pattern	Position After 3rd Search Pattern and Two Step Reductions
X_1 = length of tail	0.05000	0.04750	0.04275	0.03462
X_2 = length of section #5, insulator	0.00250	0.00275	0.00297	0.00356
X_3 = length of section #3, insulator	0.00300	0.00300	0.00300	0.00300
X_4 = length of shank	0.00300	0.00275	0.00227	0.00146
X_5 = length of head	0.05000	0.04750	0.04275	0.03462

Table 10

Example with Weight on Q_m Deviation Increased by a Factor of 100

Objective Function	Initial Point	1st Search	2nd Search	3rd Search	4th Search	5th Search
Y_1 (x) = resonance frequency	4515	4523	4539	4651	4660	5364
Y_2 (x) = ℓ_{back}/ℓ_{front}	1.31200	1.25700	1.15700	1.04600	0.89100	2.24800
Y_3 (x) = Q_m	7.36500	7.19100	6.85600	6.38800	5.74400	5.20200
Y_4 (x) = k_{eff}	0.60100	0.60300	0.60300	0.60500	0.60290	0.58900
Y_5 (x) = length	0.30800	0.30800	0.30800	0.30200	0.29400	0.27800
Y_6 (x) = weight	5.92000	5.89600	5.85600	5.68600	6.43200	4.98100
Y_7 (x) = abs (admittance)	0.00015	0.00014	0.00014	0.00013	0.00012	0.00011
Y_8 (x) = phase	0.22700	0.23100	0.24000	0.25400	0.28100	0.32500
Y_9 (x) = cost	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Y_{10}(x) = $\ell_{back}/\ell_{ceramic}$	0.51400	0.50300	0.47900	0.44900	0.39500	0.31400
Y_{11}(x) = $\ell_{front}/\ell_{ceramic}$	0.39100	0.39900	0.41400	0.42900	0.44400	0.13900

Table 10 (Continued)

Variables	Initial Point	1st Search	2nd Search	3rd Search	4th Search	5th Search
X(1) = length of tail	0.05000	0.05250	0.05725	0.06040	0.06650	0.07330
X(2) = length of section #5, insulator	0.00250	0.00225	0.00227	0.00191	0.00126	0.00070
X(3) = length of section #3, insulator	0.00300	0.00300	0.00300	0.00275	0.00227	0.00146
X(4) = length of shank	0.00300	0.00300	0.00300	0.00275	0.00252	0.00218
X(5) = length of head	0.05000	0.04750	0.04275	0.03460	0.02143	0.00065

water load is brought ever closer to the motor section, thereby increasing its effect. Simultaneously, the mass of the tail is increased to produce a larger mass loading on the back, thereby effectively clamping the back face of the ceramic. These conditions combine to load both ends of the transducer and to lower its Q_m. Note also that the |admittance| decreases while the coupling coefficient remains nearly stable.

Next, the program was rerun with the weight factor on the admittance deviation increased to 100,000, in an attempt to produce a larger value. (The weight on Q_m was decreased to its former value.) However, because of the water loading and of the initially small size of the variable perturbations (step-size EPS) used by the pattern search, an increase in the magnitude of the admittance could not be effected. The program could find no improvement over the initial calculated value of 0.15 m℧. Of course, the step reductions were not successful (only a step increase would have helped); therefore, the program executed its maximum allowed number of step reductions and submitted as a best guess the original vectors Y(I) and X(I).

When the weight factor associated with the coupling coefficient k_{eff} was increased to 10^3, the iterations shown in Table 11 resulted. Here, the final solution is not distorted very much from that for "equal" weights. This may be because the strong emphasis placed on the coupling steers the solution in the same basic direction as the combination of the other objectives does. Interestingly enough, even with such a high weighting to obtain the maximum k_{eff}, the value at the end of the 4th iteration is smaller than the 1st. This is

Table 11

Effect of Increased Weight (10^3) on k_{eff}

Objective Function	Initial Point	1st Search	2nd Search	3rd Search
Y_1 (x) = resonance frequency	4515	4589	4740	4987
Y_2 (x) = ℓ_{back}/ℓ_{front}	1.31200	1.30500	1.29500	1.25300
Y_3 (x) = Q_m	7.36500	7.24900	7.03200	6.64000
Y_4 (x) = k_{eff}	0.60130	0.60240	0.60260	0.60060
Y_5 (x) = length	0.30800	0.30300	0.29370	0.27970
Y_6 (x) = weight	5.92000	5.80600	5.58770	6.26200
Y_7 (x) = abs (admittance)	0.00015	0.00015	0.00015	0.00014
Y_8 (x) = phase	0.22700	0.22990	0.23610	0.25080
Y_9 (x) = cost	0.00000	0.00000	0.00000	0.00000
Y_{10}(x) = $\ell_{back}/\ell_{ceramic}$	0.51400	0.51160	0.50710	0.49370
Y_{11}(x) = $\ell_{front}/\ell_{ceramic}$	0.39170	0.39180	0.39150	0.39390

Table 11 (Continued)

Variables	Initial Point	1st Search	2nd Search	3rd Search
X(1) = length of tail	0.05000	0.04750	0.04275	0.03710
X(2) = length of section #5, insulator	0.00250	0.00275	0.00297	0.00331
X(3) = length of section #3, insulator	0.00300	0.00300	0.00300	0.00300
X(4) = length of shank	0.00300	0.00275	0.00227	0.00171
X(5) = length of head	0.05000	0.04750	0.04275	0.03462

probably caused by the influence of the other objective functions and the limited number of searches.

A last example was set in which the weight on the length deviation was increased to 1,000. As in the previous case for k_{eff}, the goal of minimum length must be compatible with the other objectives because through four searches the solution was identically equal to that of "equal" weights.

4.3 The Question of Uniqueness

There are three separate phenomena, each of which are multimodal, which obviate the possibility of a unique solution. Firstly, the piezoceramic motor section of the transducer has an infinity of resonances in any direction of vibration. Secondly, the addition of variable masses for the head and tail provides an infinity of combinations to produce any specific resonance frequency. Addition of other variable masses (insulators, shank, and acoustic window) serve to increase the infinity of combinations which have the same resonance frequency. While the resonance frequency isn't the only objective function, its deviation variables will dominate the achievement function in the first few iterations. Thirdly, any nonlinear optimization technique is multimodal and the achievement function reached in the NLGP technique is typically a function of the starting point, the step sizes, and the order of ranking of the independent variables.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The broad objective of this thesis was to develop an accurate tool for transducer design. A modified Mason's Equivalent Circuit was used to construct a mathematical model (Figure 7) which suitably predicts the design parameters of resonance frequency, admittance magnitude and phase, and node placement. Nonlinear Goal Programming was selected as a viable multiobjective optimization technique to evaluate the mathematical model, compare the results with the specified design parameters, and provide the optimal compromise values for the design variables.

In Chapter 1, the basic transducer design problem was presented. Transducer theory was explored in Chapter 2 where equations which relate the design variables to the transducer responses were developed. The reliability of the equations and of the computer algorithm was examined with comparisons between both theory and experiment. Chapter 3 contained a description of Nonlinear Goal Programming and its application to the transducer model. A hypothetical transducer was designed in Chapter 4 to illustrate NLGP operation and to display the importance of the weight factor.

5.2 Conclusions

Goal programming is a powerful technique for obtaining the optimal compromise solution to complex multiobjective problems. "Tonpilz" transducer design, involving all segments of the body of the transducer and its associated housing, can easily be accomplished using this technique, but it must be recognized that as with any other nonlinear optimization tool, or any multimodal system, the resultant solution may well be only a local optimal solution. Further research should be done to create a theoretical model to explain the frequency differences encountered by actual "identical" transducers, which caused the errors outlined in Chapter 3.

One of the more exciting aspects of this design program is that it provides an opportunity for theoretical research into the behavior of all types of transducers. The optimization technique allows any combination of attributes to be examined in relation to the problem variables as has never before been possible. Finally, this technique can easily be applied to the design and analysis of other types of systems and to other optimization problems. It is hoped that this thesis might guide the researcher in other areas.

APPENDIX A

CALCULATION OF WAVE-GUIDE EQUATION (8) FROM LOADED T-SECTION

Figure 13 is a T circuit representation of a wave guide terminated with a load impedance Z_L. The input impedance can be seen to be:

$$Z_{in} = \frac{(Z_L + jZ_0\tan(x/2))\ (-jZ_0/\sin(x))}{Z_L + jZ_0\tan(x/2) - jZ_0/\sin(x)} + jZ_0\tan(x/2)\ , \qquad (20)$$

where $x = k_i \ell_i$.

Using a trigonometric substitution,

$$\tan(x/2) - 1/\sin(x) = -\cot(x) \qquad (21)$$

$$Z_{in} = \frac{-jZ_0 Z_L\cot(x) + Z_0^2\tan(x/2)\ (2/\sin(x) - \tan(x/2))}{Z_L - jZ_0\cot(x)} \qquad (22)$$

but

$$2/\sin(x) - \tan(x/2) = 1/\tan(x/2)\ . \qquad (23)$$

Therefore,

$$Z_{in} = \frac{-jZ_0 Z_L\cot(x) + Z_0^2}{Z_L - jZ_0\cot(x)}\ , \qquad (24)$$

which is expressed in the text as:

$$Z_{in} = \frac{Z_0(Z_L + jZ_0\tan(x))}{Z_0 + jZ_L\tan(x)} \qquad (25)$$

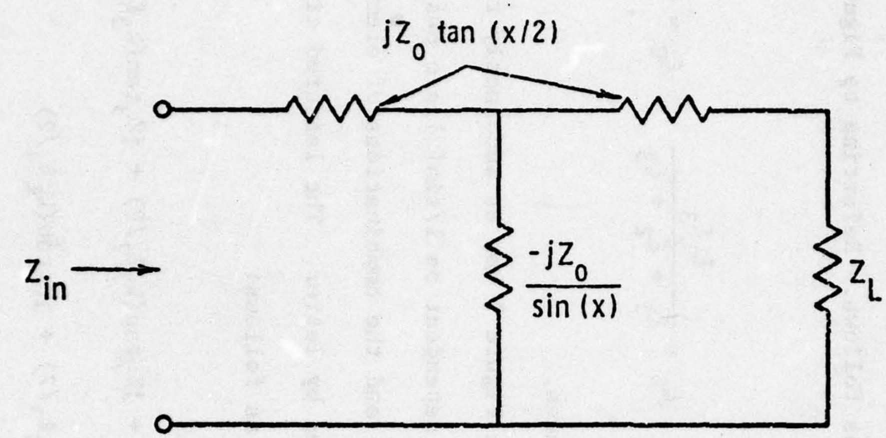


Figure 13. Circuit Diagram for a Loaded T-Section.

APPENDIX B

SOLUTION OF TRANSDUCER CIRCUIT WHICH INCLUDES A STRESS BOLT

The method of analysis used to simplify the transducer circuit is a π-T transformation as follows. Referring to Figure 14, one can equate:

$$\xi_A = \frac{\xi_1 \xi_2}{\xi_1 + \xi_2 + \xi_3} \qquad \xi_B = \frac{\xi_1 \xi_3}{\xi_1 + \xi_2 + \xi_3} \qquad \xi_C = \frac{\xi_2 \xi_3}{\xi_1 + \xi_2 + \xi_3} \tag{26}$$

where the ξ's are impedances.

The circuit shown in Figure 7 can be succinctly reduced to Figure 15. The elements dependent on 1/sin(x) are designated by number as assigned in Figure 12, and the combinations of elements dependent on tan(x/2) are designated by letter. The lettered circuit elements of Figure 15 are defined as follows:

$$\begin{aligned}
Z_A &= Z[\leftarrow_{head}] + jZ_7\tan(k_7\ell_7/2) + jZ_3\tan(k_3\ell_3/2) \\
Z_B &= jZ_3\tan(k_3\ell_3/2) + jZ_4\tan(k_4\ell_4/2) \\
Z_C &= jZ_4\tan(k_4\ell_4/2) + jZ_5\tan(k_5\ell_5/2) \\
Z_D &= jZ_5\tan(k_5\ell_5/2) + jZ_6\tan(k_6\ell_6/2) \\
Z_E &= jZ_6\tan(k_6\ell_6/2) + jZ_7\tan(k_7\ell_7/2)
\end{aligned} \tag{27}$$

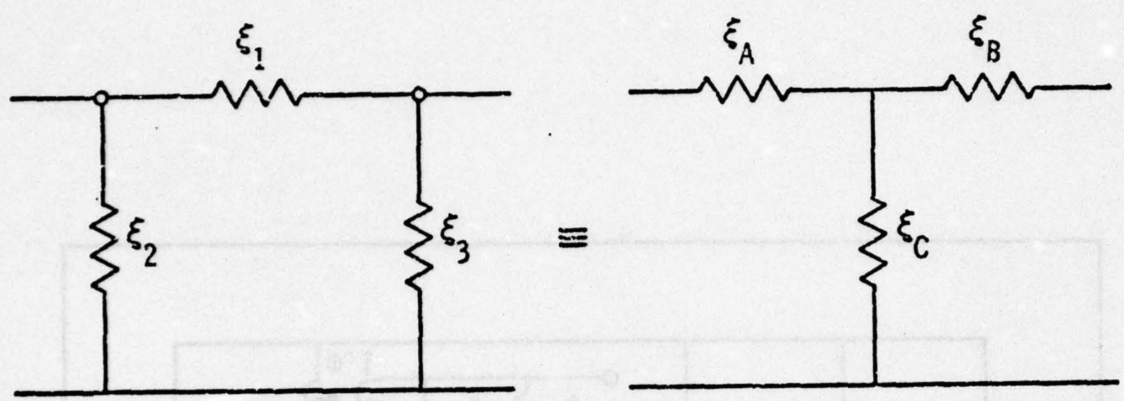


Figure 14. π-T Transformation.

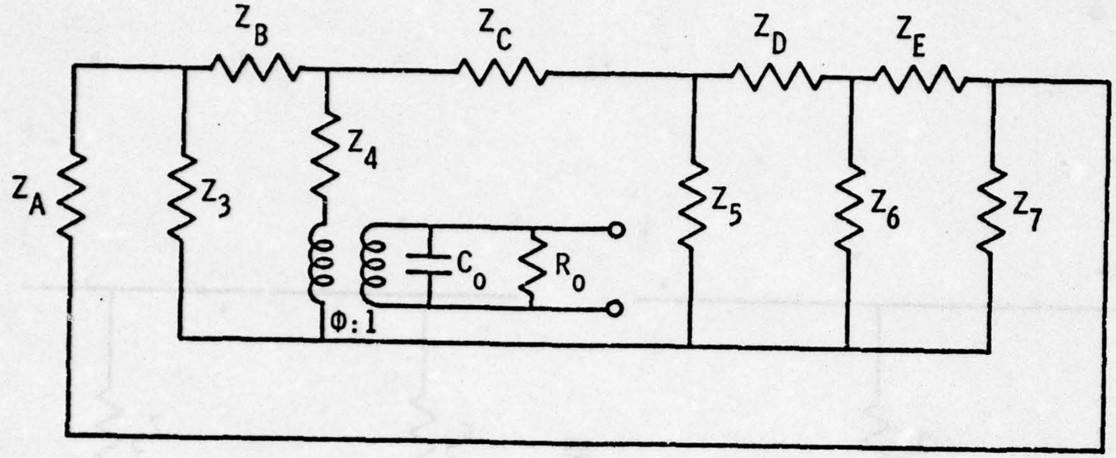


Figure 15. Compressed Transducer Circuit.

Redrawing the circuit from Figure 15, one obtains Figure 16(a). Applying the π-T transformation to the circled segment yields Figure 16(b). The newly defined elements from this operation are:

$$Z_{F1} = \frac{Z_E Z_7}{Z_E + Z_7 + Z_6} \qquad Z_{F2} = \frac{Z_E Z_6}{Z_E + Z_6 + Z_7} \qquad Z_{F3} = \frac{Z_6 Z_7}{Z_E + Z_6 + Z_7} \tag{28}$$

As a result of this operation, some elements are in series and can be readily combined. Let

$$Z_{A1} = Z_{F1} + Z_A \qquad Z_{D2} = Z_{F2} + Z_D$$

Following this combination, the circuit is redrawn as shown in Figure 17. The process is repeated for the new circled π-section, causing the following variables to be defined:

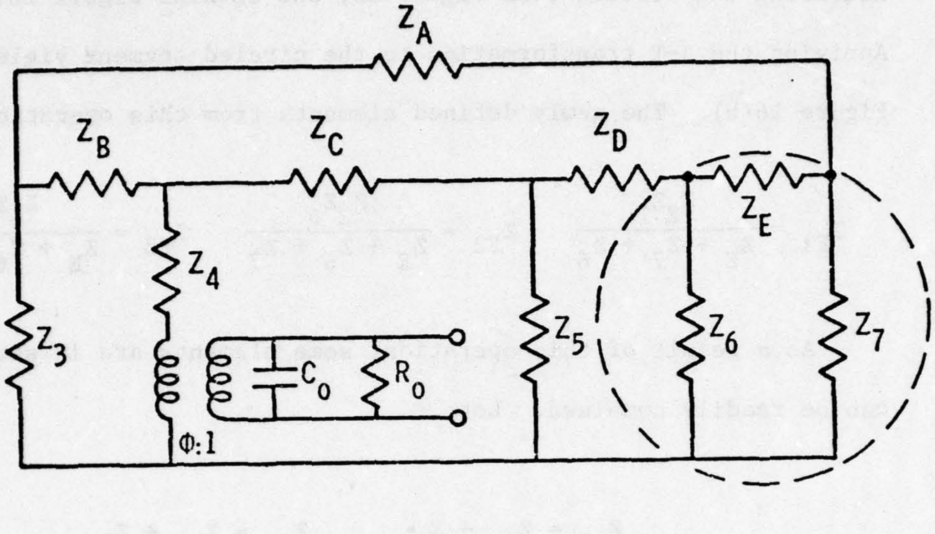
$$Z_{G1} = \frac{Z_{D2} Z_{F3}}{Z_{D2} + Z_{F3} + Z_5} \qquad Z_{G2} = \frac{Z_{D2} Z_5}{Z_{D2} + Z_{F3} + Z_5} \qquad Z_{G3} = \frac{Z_5 Z_{F3}}{Z_{D2} + Z_{F3} + Z_5}$$

$$Z_{A2} = Z_{G1} + Z_{A1} \qquad Z_{C1} = Z_{G2} + Z_C \tag{29}$$

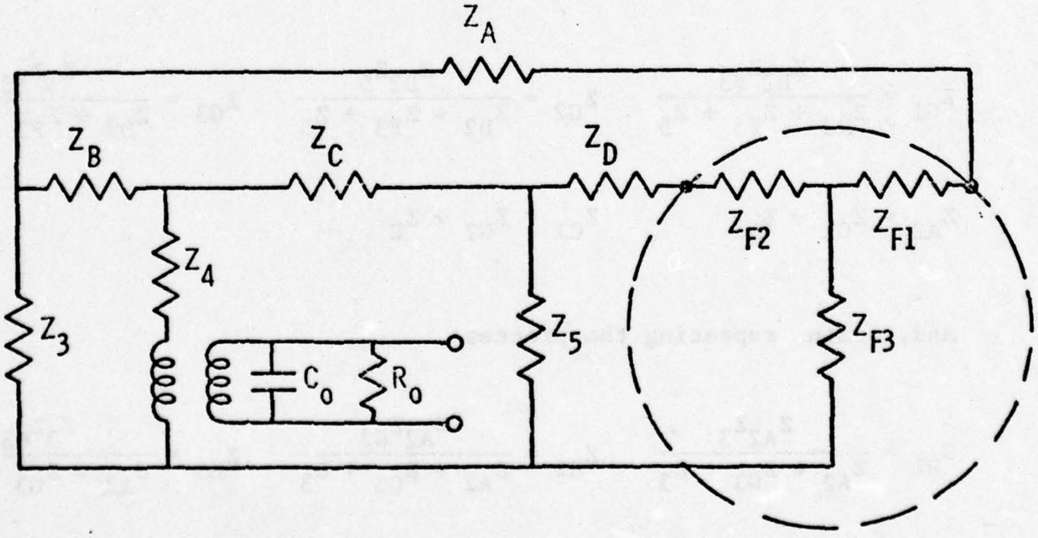
And, again, repeating the process:

$$Z_{H1} = \frac{Z_{A2} Z_3}{Z_{A2} + Z_{G3} + Z_3} \qquad Z_{H2} = \frac{Z_{A2} Z_{G3}}{Z_{A2} + Z_{G3} + Z_3} \qquad Z_{H3} = \frac{Z_3 Z_{G3}}{Z_{A2} + Z_{G3} + Z_3}$$

$$Z_{C2} = Z_{H2} + Z_{C1} \qquad Z_{B1} = Z_{H1} + Z_B \tag{30}$$



(a) Compressed Transducer Circuit with π-Section Circled



(b) π-T Transformation Applied

Figure 16. π-T Transformation.

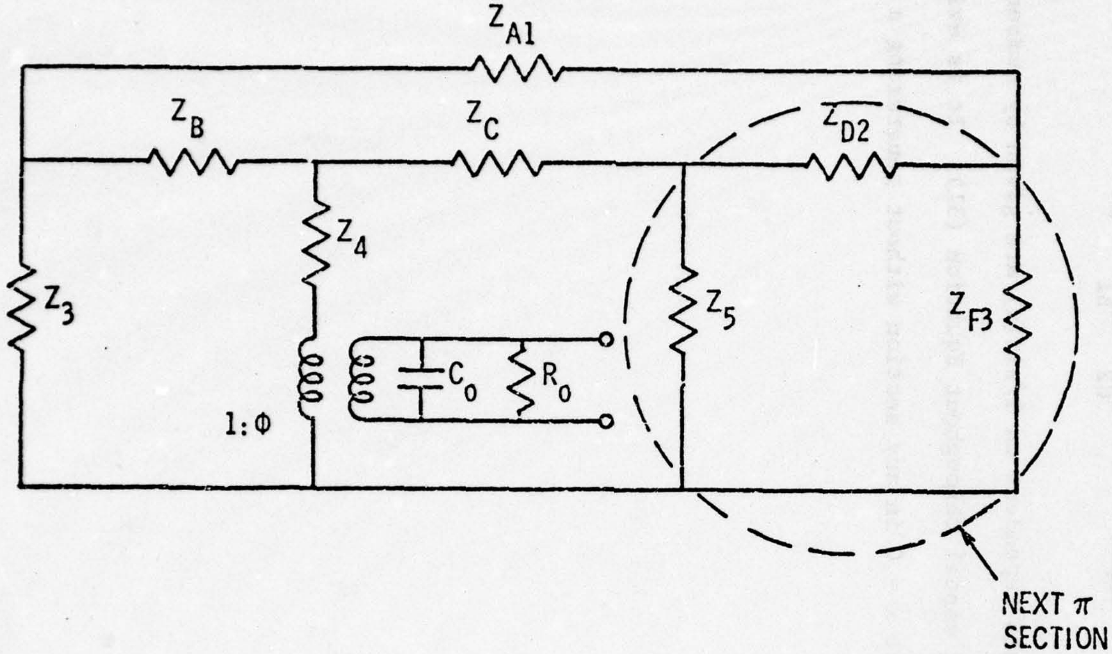


Figure 17. Reduced Transducer Circuit as a Result of One Application of a π-T Transformation.

The circuit has now been reduced to that of Figure 18. From this, one can calculate the mechanical impedance which is:

$$Z_{input} = Z_4 + \frac{Z_{C2}Z_{B1}}{Z_{C2} + Z_{B1}} + Z_{H3} \tag{31}$$

All circuit elements dependent on $1/\sin(x)$ are given by number, and these terms do not cancel throughout Equation (31). It is evident that one cannot let $x = 0$ in any section without generating a division by zero in Z_{input}.

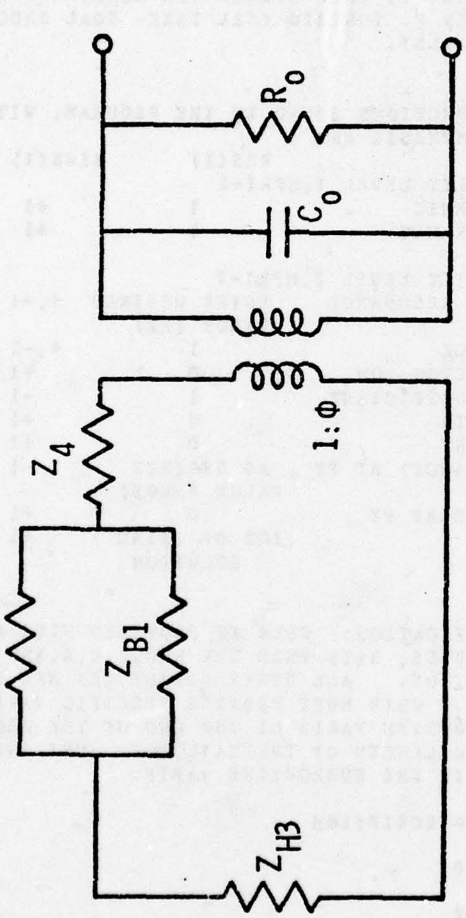


Figure 18. Final Reduction of Circuit.

APPENDIX C

PROGRAM

```
C NLGP TRANSDUCER DESIGN                    JULY, 1979
C      THIS PROGRAM IS FOR TRANSDUCER DESIGN.  ITS PURPOSE IS TO
C  PROVIDE A METHOD FOR DESIGNING A SPECIFIC TONPILZ TRANSDUCER GIVEN
C  THE DESIRED SPECIFICATIONS.  THE ANALYST SUPPLIES THE RESPONSES
C  DESIRED OF THE TRANSDUCER, THE VARIABLES, AND AN INITIAL TRIAL
C  SOLUTION.  THE PROBLEM IS THEN SOLVED VIA NONLINEAR GOAL PROGRAMMING-
C  AS DEVELOPED BY JAMES P. IGNIZIO (SEE TEXT- GOAL PROGRAMMING AND
C  EXTENSIONS, FOR DETAILS).
C
C
C      THE OBJECTIVE FUNCTIONS FOUND IN THE PROGRAM, WITH THE SUGGESTED
C  WEIGHTS FOR EQUAL EMPHASIS ARE :
C  IROW=I        Y(I)                 RHS(I)       SIGN(I)  WEIGHT(I)
C                  PRIORITY LEVEL 1,MPRI=1
C   10       LBACK/LCERAMIC                1           +1     1
C   11       LFRONT/LCERAMIC               1           +1     1
C
C                  PRIORITY LEVEL 2,MPRI=2
C    1       ADMITTANCE RESONANCE     ENTER DESIRED  +,-1    0.1
C                                     VALUE (HZ)
C    2       LFRONT/LBACK                  1         +,-1    10
C    3       QUALITY FACTOR, QM            0           +1     1
C    4       COUPLING COEFFICIENT          1           -1    10
C    5       TOTAL LENGTH                  0           +1    10
C    6       TOTAL WEIGHT                  0           +1     1
C    7       ABS(ADMITTANCE) AT FY   AS DESIRED        -1    1000
C                                    VALUE (MHOS)
C    8       PHASE ANGLE AT FY             0           +1    100
C    9       TOTAL COST              20% OF TRIAL      +1     1
C                                       SOLUTION
C
C
C      VARIABLE IDENTIFICATION:  USER IS PROVIDED WITH A CHOICE
C  OF AT MOST 12 VARIABLES, X(I) FROM THE RHO,C,L,A,AND ETA OF
C  EACH WAVE GUIDE SECTION.   ALL OTHER PARAMETERS ARE FIXED AT
C  INITIAL DATA POINTS.  USER MUST PROVIDE SPECIFIC VARIABLE
C  ASSIGNMENTS IN SUBROUTINE VARIB AT THE END OF THE DECK.
C  (EG. TO OPTIMISE THE LENGTH OF THE TAIL, ONE MUST INTRODUCE A
C  CARD  L(6) = X(1)  IN THE SUBROUTINE VARIB.
C
C  WAVE GUIDE SECTION DESCRIPTION
C      1  HEAD
C      2  SHANK OF HEAD
C      3  INSULATOR
C      4  CERAMIC STACK
C      5  INSULATOR
C      6  TAIL
C      7  RUBBER WINDOW
C      8  PRESSURE RELIEF
C
C  NOTE: TO ELIMINATE ANY SECTION, SET THE LENGTH =0.0 IN THE
C  DATA INPUT.  THE CORRESPONDING RHO,C, AND A CAN EQUAL 1.0
C
```

```
C***************************************************************************
C   VARIABLES                  UTILITY
C       NVAR          NUMBER OF DECISION VARIABLES
C       NOBJ          NUMBER OF OBJECTIVE FUNCTIONS
C       NPRIOR        NUMBER OF PRIORITY LEVELS
C       NACH          NUMBER OF TERMS IN ACHIEVEMENT FUNCTION
C       NMAX          MAXIMUM NUMBER OF PATTERN SEARCH CYCLES ALLOWED
C       X             VECTOR OF DECISION VARIABLES AT TEST POINT
C       EPS           VECTOR OF STEP SIZES USED IN PATTERN SEARCH
C       RHS           RIGHT HAND SIDE VALUES OF OBJECTIVE FUNCTIONS
C       MPRI          PRIORITY LEVEL OF GIVEN DEVIATION VARIABLE
C       WEIGHT        WEIGHT FACTOR OF GIVEN DEVIATION VARIABLE
C       ALPHA         PATTERN SEARCH ACCELERATION FACTOR
C       BETA          STEP REDUCTION FACTOR
C       MAXRED        MAXIMUM NUMBER OF STEP REDUCTIONS ALLOWED
C       IPRINT        OUTPUT SWITCH--0 GIVES ONLY FINAL RESULT
C                     1 GIVES RESULT OF EVERY PATTERN SEARCH CYCLE
C       EPSY          VECTOR OF IMPROVEMENT TOLERANCES IN ACHIEVEMENT
C       SIGN          INDICATES POSITIVE OR NEGATIVE DEVIATION
C                     TO BE MINIMIZED FROM OBJECTIVE FUNCTION IROW AT
C                     MPRI LEVEL WITH WEIGHT FACTOR WEIGHT
C       IROW          OBJECTIVE FUNCTION OF GIVEN DEVIATION VARIABLE
C       Y             VECTOR OF FUNCTIONS OF DECISION VARIABLES
C       XBASE         BASE POINT FOR HOOKE-JEEVES ALGORITHM
C       ZTEST         ACHIEVEMENT FUNCTION VALUE VECTOR AT A TEST POINT
C       RHO           VECTOR OF DENSITY VALUES (MKS) KG/CU M
C       C             VECTOR OF SOUND SPEEDS M/SEC
C       L             LENGTH OF EACH WAVE GUIDE SECTION, M
C       A             AREA OF EACH SECTION SQUARE METERS
C       ETA            VECTOR OF LOSS FACTORS
C       TURNS          THE SQUARE OF THE TRANSFORMATION FACTOR
C       CAP            THE CALCULATED CAPACITANCE
C       ZW             THE LOADING PRESENTED TO THE FRONT FACE
C       ZPR            THE LOADING PRESENTED BY THE WEB APPLIED TO THE
C                      PRESSURE RELIEF
C       STEP           THE FREQUENCY INCREMENTS USED TO FIND RESONANCE
C                      BY ITERATION
C       COST           VECTOR OF VALUES OF EACH COST PER UNIT VOULME
C*************USERS GUIDE TO INPUT DATA**************************************
C   VARIABLE                                              FORMAT
C   CARD 1
C     NVAR,NOBJ,NPRIOR,NACH,NMAX                          5I5
C   CARD 2
C     IPRINT,MAXRED                                       2I5
C   CARD(S) 3-
C     X(I),I=1,NVAR                                       8E10.4
C   CARDS(S) 4-
C     EPS(I),I=1,NVAR                                     8E10.4
C   CARD(S) 5-
C     RHS(I),I=1,NOBJ                                     8E10.4
C    CARD 6
C     ALPHA,BETA                                          2E10.4
C    CARD 7
```

```
C        EPSY(I),I=1,NPRIOR                                   8E10.4
C     CARDS 8-22
C        SIGN(I),IROW(I),MPRI(I),WEIGHT(I),I=1,NACH          F5.0,2I5,F10.0
C     CARDS 23-24
C        COST(I),I=1,8                                        8E10.4
C     CARDS 25-34
C        RHO(I),C(I),L(I),A(I),ETA(I),I=1,8                  8E10.4
C     CARD 35
C        TURNS,CAP,ZW,ZPR                                     6E10.4
C        NOTE: ZW AND ZPR ARE COMPLEX AND REQUIRE 2 DATA FIELDS EACH
C     CARD 36
C        STEP                                                 I5
C
C
C***********************************************************************
C
C       INFORMATION ON DIMENSION STATEMENTS
C
C       X(I), I = TOTAL NUMBER OF VARIABLES (NVAR).
C       Z(I), I = TOTAL NUMBER OF PRIORITIES (NPRIOR)
C       EPS(I), I = TOTAL NUMBER OF VARIABLES (NVAR).
C       RHS(I), I = TOTAL NUMBER OF OBJECTIVES (NOBJ)
C       EPSY(I), I = TOTAL NUMBER OF PRIORITIES (NPRIOR)
C       SIGN(I), I = TOTAL NUMBER OF TERMS IN ACHIEVEMENT FUNCTION (NACH)
C       IROW(I), I = TOTAL NUMBER OF TERMS IN ACHIEVEMENT FUNCTION (NACH).
C       MPRI(I), I = TOTAL NUMBER OF TERMS IN ACHIEVEMENT FUNCTION (NACH)
C       WEIGHT(I), I = TOTAL NUMBER OF TERMS IN ACHIEVEMENT
C          FUNCTION (NACH).
C       Y(I), I = TOTAL NUMBER OF OBJECTIVES (NOBJ).
C*****************************************************************
        DIMENSION X(12),Y(12)
        COMMON/COMM01/NVAR,NOBJ,NPRIOR,NACH,NMAX
        COMMON/COMM02/X
        COMMON/COMM07/Y
        CALL DATAIN
C          ALL NECESSARY DATA IS INPUT AND PRINTED OUT
        CALL HJALG
C          THE PATTERN SEARCH ALGORITHM
        PRINT 600
        PRINT 601
        CALL VARIB
        CALL YVALUE
        DO 1 K=1,NOBJ
        PRINT 602,K,Y(K)
1       CONTINUE
        PRINT 603
        DO 2 K=1,NVAR
        PRINT 604,K,X(K)
2       CONTINUE
600     FORMAT(1H0,25X,'MAXIMUM NUMBER OF PATTERN SEARCH CYCLES EXCEEDED',
       $/,1H ,25X,'BEST SOLUTION TO THIS POINT FOLLOWS')
601     FORMAT(1H0,25X,' OBJECTIVE FUNCTION VALUES')
602     FORMAT(1H ,25X,'Y(',I2,') =  ',F13.6)
```

```
603   FORMAT(1H0,25X,'DECISION VARIABLES VALUES')
604   FORMAT(1H ,25X,'X(',I2,') = ',F13.6)
      STOP
      END
      SUBROUTINE DATAIN
      COMPLEX ZW,ZPR
      REAL L(9)
      INTEGER STEP
      DIMENSION X(12),EPS(12),RHS(12),EPSY(3),SIGN(24),IROW
     $(24),MPRI(24),WEIGHT(24),RHO(9),C(9),A(9),ETA(9),COST(9)
      COMMON/COMM01/NVAR,NOBJ,NPRIOR,NACH,NMAX
      COMMON/COMM02/X
      COMMON/COMM03/EPS
      COMMON/COMM04/RHS
      COMMON/COMM05/SIGN,WEIGHT,IROW,MPRI
      COMMON/COMM06/EPSY
      COMMON/COMM12/RHO,C,L,A,ETA
      COMMON/COMM13/TURNS,CAP,ZW,ZPR,STEP,FY,COST
      COMMON/COMM09/ALPHA,BETA
      COMMON/COMM10/MAXRED
      COMMON/COMM11/IPRINT
      READ 500,NVAR,NOBJ,NPRIOR,NACH,NMAX
      PRINT 600, NVAR,NOBJ,NPRIOR,NACH,NMAX
      READ 500,IPRINT,MAXRED
      PRINT 612,IPRINT,MAXRED
      READ 501,(X(I),I=1,NVAR)
      PRINT 602
      DO 2 I=1,NVAR
      PRINT 601,I,X(I)
    2 CONTINUE
      READ 501,(EPS(I),I=1,NVAR)
      PRINT 603
      DO 3 I=1,NVAR
      PRINT 604,I,EPS(I)
    3 CONTINUE
      PRINT 800
      READ 501,(RHS(I),I=1,NOBJ)
      PRINT 605
      DO 4 I=1,NOBJ
      PRINT 606,I,RHS(I)
    4 CONTINUE
      FY= RHS(1)
      READ 501, ALPHA,BETA
      PRINT 607,ALPHA,BETA
      READ 501,(EPSY(I),I=1,NPRIOR)
      PRINT 608
      DO 5 I=1,NPRIOR
      PRINT 609,I,EPSY(I)
    5 CONTINUE
      PRINT 610
      DO 1 N=1,NACH
```

```
      READ 502,SIGN(N),IROW(N),MPRI(N),WEIGHT(N)
      PRINT 611,SIGN(N),IROW(N),MPRI(N),WEIGHT(N)
    1 CONTINUE
      PRINT 701
      READ 501, (COST(I),I=1,8)
      DO 20 N=1,8
      READ 501,RHO(N),C(N),L(N),A(N),ETA(N)
      PRINT 700,N,RHO(N),C(N),L(N),A(N),ETA(N),COST(N)
   20 CONTINUE
      PRINT 702
      READ 501,TURNS,CAP,ZW,ZPR
      PRINT 703,TURNS,CAP,ZW,ZPR
      READ 500,STEP
      PRINT 704, STEP
  500 FORMAT(16I5)
  501 FORMAT(8E10.4)
  502 FORMAT(F5.0,2I5,F10.0)
  700 FORMAT(15X,I2,6E15.4)
  701 FORMAT(1H1,22X,'RHO',15X,'C',14X,'L',14X,'A',13X,'ETA',13X,'COST')
  702 FORMAT(/,25X,' TURNS RATIO',9X,'CAP',7X,'RAD LOAD',15X,'WEB LOAD')
  703 FORMAT(25X,E10.4,5X,E10.4,4X,2E10.4,3X,2E10.4)
  704 FORMAT(/,25X,' FREQUENCY LOOP STEP SIZE',I10,///)
  600 FORMAT(1H ,25X,'NUMBER OF DECISION VARIABLES =',T68,I5,/,1H ,25X,'
     $NUMBER OF OBJECTIVE FUNCTIONS =',T68,I5,/,1H ,25X,'NUMBER OF PRIOR
     $ITY LEVELS =',T68,I5,/,1H ,25X,'NUMBER OF TERMS IN ACHIEVEMENT FUN
     $CTION =',T68,I5,/,1H ,25X,'MAXIMUM NUMBER OF SEARCH PATTERNS =',T6
     $8,I5,/)
  612 FORMAT(1H ,' IPRINT',I5,25X,' MAXIMUM NUMBER REDUCTIONS',T68,I5)
  601 FORMAT(1H ,25X,'X(',I2,')  =  ',E10.4)
  602 FORMAT(1H1,25X,'VECTOR OF INITIAL GUESSES OF DECISION VARABLES --
     $X')
  603 FORMAT(1H1,25X,'INITIAL INCREMENTS FOR DECISION VARIABLES -- EPS')
  604 FORMAT(1H ,25X,'EPS(',I2,')  =  ',F10.6)
  605 FORMAT(1H1,25X,'VALUE OF RIGHT-HAND SIDE OF OBJECTIVE FUNCTIONS')
  606 FORMAT(1H ,25X,'RHS(',I2,')  =  ',F10.4)
  607 FORMAT(1H1,25X,'ALPHA = ',F8.4,10X,'BETA = ',F8.4)
  608 FORMAT(1H1,25X,'VECTOR OF ACHIEVEMENT FUNCTION TOLERANCES')
  609 FORMAT(1H ,25X,'EPSY(',I2,')  =  ',F9.7)
  610 FORMAT(1H1,T25,'DEVIATION TYPE',T45,'OBJECTIVE NUMBER',T65,'PRIORI
     $TY LEVEL',T85,'WEIGHT FACTOR')
  611 FORMAT(1H ,T30,F6.1,T54,I2,T74,I2,T91,F8.2)
  800 FORMAT(1H1,25X,'OBJECTIVE FUNCTIONS',/,25X,'Y(1) = FY',/,25X,'Y(2)
     1 = LBACK/LFRONT',/,25X,'Y(3) = QY',/,25X,'Y(4) = KEFF',/,25X,'Y(5)
     2 = LENGTH',/,25X,'Y(6) = WEIGHT',/,25X,'Y(7) = ABS(ADMIT)',/,25X,
     3'Y(8) = PHASE OF ADMIT',/,25X,'Y(9) = COST',/,25X,'Y(10) = LBACK/
     4LCERAMIC',/,25X,'Y(11) = LFRONT/LCERAMIC')
      RETURN
      END
      SUBROUTINE VALUE(Z)
C        SUBROUTINE VALUE EVALUATES THE ACHIEVEMENT FUNCTION GIVEN THE
C        PREVIOUSLY EVALUATED FUNCTIONS OF THE DECISION VARIABLES
      DIMENSION Z(3),Y(12),RHS(12),SIGN(24),IROW(24),MPRI(24),WEIGHT(24)
      COMMON/COMM01/NVAR,NOBJ,NPRIOR,NACH,NMAX
```

```
      COMMON/COMM04/RHS
      COMMON/COMM05/SIGN,WEIGHT,IROW,MPRI
      COMMON/COMM07/Y
      CALL VARIB
      CALL YVALUE
      DO 1 K=1,NPRIOR
      Z(K)=0.0
1     CONTINUE
      DO 2 K=1,NACH
      IF(SIGN(K).LT.0.0)GOTO4
      DEV=Y(IROW(K))-RHS(IROW(K))
      GOTO5
4     DEV=RHS(IROW(K))-Y(IROW(K))
5     IF(DEV.LT.0.0)DEV=0.0
      Z(MPRI(K))=Z(MPRI(K))+WEIGHT(K)*DEV
2     CONTINUE
      RETURN
      END
      SUBROUTINE HJALG
      DIMENSION XSAVE(12),Y(12)
      DIMENSION X(12),XBASE(12)
      DIMENSION EPS(12),Z(3)
      COMMON/COMM01/NVAR,NOBJ,NPRIOR,NACH,NMAX
      COMMON/COMM02/X
      COMMON/COMM03/EPS
      COMMON/COMM07/Y
      COMMON/COMM08/Z
      COMMON/COMM09/ALPHA,BETA
      COMMON/COMM10/MAXRED
      COMMON/COMM11/IPRINT
C        INITIALIZATION OF PATTERN SEARCH
      MCOUNT=0
C     MCOUNT COUNTS THE NUMBER OF PATTERN SEARCHES ATTEMPTED
      NREDUC=0
C        NREDUC COUNTS THE NUMBER OF STEP SIZE REDUCTIONS

      KFLAG=1
C        KFLAG IS A SWITCH WHICH TELLS THE PROGRAM WHETHER THE NEW TEST
C        POINT FOUND BY ACCELERATION IMPROVES THE VALUE OF THE ACHIEVE-
C        MENT FUNCTION
100   CALL VALUE(Z)
      PRINT 601
      DO 30 K=1,NOBJ
      PRINT 602,K,Y(K)
 30   CONTINUE
      PRINT 603
101   DO 1 K=1,NVAR
      XBASE(K)=X(K)
      PRINT 604,K,X(K)
1     CONTINUE
C        START THE SEARCH PATTERN
199   NIJK=0
C        NIJK COUNTS THE NUMBER OF IMPROVEMENTS,IF ANY, DURING A PATTERN
C        SEARCH
200   DO 3 K=1,NVAR
```

```
C        STEP FORWARD AND EVALUATE
      X(K)=X(K)+EPS(K)
C        SUBROUTINE DECIDE TAKES THE VALUE OF THE TEST POINT, EVALUATES
C        THE ACHIEVEMENT FUNCTION VIA ZVALUE, AND DECIDES IF THE TEST
C        POINT IS AN IMPROVEMENT. IJK=1 FOR IMPROVEMENT, IJK=0 IF NOT
      CALL DECIDE(IJK)
      NIJK=NIJK+IJK
      IF(IJK.EQ.1) GO TO 3
C        STEP BACKWARDS AND EVALUATE
      X(K)=X(K)-2.0*EPS(K)
      CALL DECIDE(IJK)
      NIJK=NIJK+IJK
      IF(IJK.EQ.1) GO TO 3
      X(K)=X(K)+EPS(K)
3     CONTINUE
      MCOUNT=MCOUNT+1
C        CHECK IF MAXIMUM NUMBER OF SEARCH CYCLES EXHAUSTED
      IF(MCOUNT.EQ.NMAX)GOTO999
C        CHECK TO SEE IF PATTERN RESULTED IN NO IMPROVEMENT
      IF(NIJK.EQ.0.AND.KFLAG.EQ.1)GOTO300
C        ACCELERATION
      DO 10 K=1,NVAR
      XSAVE(K)=X(K)
10    CONTINUE
      DO 11 K=1,NVAR
      X(K)=X(K)+ALPHA*(X(K)-XBASE(K))
11    CONTINUE
      DO 12 K=1,NVAR
      XBASE(K)=XSAVE(K)
12    CONTINUE
C        NOW CHECK VALUE OF ACHIEVEMENT FUNCTION AT ACCELERATION POINT
      KFLAG=0
      CALL DECIDE(IJK)
      IF(IJK.EQ.0)KFLAG=1
      IF(IPRINT.NE.0)PRINT 600,MCOUNT
      IF(KFLAG.EQ.1) GO TO 17
      PRINT 601
      DO 14 K=1,NOBJ
      PRINT 602,K,Y(K)
14    CONTINUE
      PRINT 603
      DO 15 K=1,NVAR
      PRINT 604,K,X(K)
15    CONTINUE
      PRINT 606
      GO TO 199
17    CONTINUE
      PRINT 607
13    GOTO199
C        REDUCE STEP SIZE
300   NREDUC=NREDUC+1
C        IF MAXIMUM NUMBER OF STEP SIZE REDUCTIONS EQUALED, TERMINATE
      DO 301 K=1,NVAR
      X(K)=XBASE(K)
```

```
301     CONTINUE
        IF(NREDUC.GT.MAXRED) GO TO 316
304     DO 305 K=1,NVAR
        EPS(K)=EPS(K)*BETA
305     CONTINUE
        PRINT 605,NREDUC,MCOUNT
        GOTO199
316     PRINT 608
        PRINT 601
        CALL VARIB
        CALL YVALUE
        DO 318 K=1,NOBJ
        PRINT 602,K,Y(K)
318     CONTINUE
        PRINT 603
        DO 319 K=1,NVAR
        PRINT 604,K,XBASE(K)
319     CONTINUE
317        STOP
600     FORMAT(1H1,25X,I3,' SEARCH PATTERNS HAVE BEEN COMPLETED')
601     FORMAT(1H1,25X,' OBJECTIVE  FUNCTION')
602     FORMAT(1H ,25X,'Y(',I2,')  =  ',F13.6)
603     FORMAT(1H1,25X,'DECISION VARIABLES')
604     FORMAT(1H ,25X,'X(',I2,')  =  ',F13.6)
605     FORMAT(1H1,25X,'STEP REDUCTION NUMBER ',I3,' OCCURRED AFTER SEARCH
       $ PATTERN NUMBER ',I3)
606     FORMAT(1H ,25X,'KFLAG = 0 SO THAT THE TEMPORARY BASE POINT IMPROVE
       $S THE SOLUTION')
607     FORMAT(1H ,25X,'KFLAG = 1 SO THAT THE TEMPORARY BASE POINT DOES NO
       $T IMPROVE THE SOLUTION',/,1H ,25X,'SHOULD THE PATTERN SEARCH FIND
       $NO IMPROVEMENT, STEP SIZES WILL BE REDUCED')
608     FORMAT(1H1,25X,'PATTERN SEARCH TERMINATED -- MAXIMUM NUMBER OF
       $STEP REDUCTIONS EQUALED')
999     RETURN
        END

        SUBROUTINE DECIDE(M)
C          SUBROUTINE DECIDE CALLS SUBROUTINE VALUE TO EVALUATE THE ACH-
C          IEVEMENT FUNCTION AT THE TEST POINT AND COMPARES THIS TO THE
C          PREVIOUS VALUE.  IF THE POINT WILL PROVIDE IMPROVEMENT, M=1
C          IS RETURNED TO THE ARGUMENT LIST, IF NOT M=0 IS RETURNED.
        DIMENSION X(12)
        DIMENSION Z(3),ZTEST(3),EPSY(3),Y(12)
        COMMON/COMM01/NVAR,NOBJ,NPRIOR,NACH,NMAX
        COMMON/COMM02/X
        COMMON/COMM06/EPSY
        COMMON/COMM07/Y
        COMMON/COMM08/Z
        CALL VALUE(ZTEST)
        DO 10 K=1,NPRIOR
        IF(ZTEST(1).GT.Z(1))GO TO 12
        IF(ZTEST(K).LT.Z(K)) GO TO 11
10      CONTINUE
        GOTO12
 11     DO 16 K=1,NPRIOR
C          TERMINATION BY MEANS OF ZERO FOR ALL PRIORITY LEVELS
        IF(ZTEST(K).NE.0.0)GOTO17
```

```
16      CONTINUE
        GOTO999
17      DO 13 K=1,NPRIOR
C          TERMINATION TEST BY TOLERANCES
        IF(ABS(Z(K)-ZTEST(K)).GT.EPSY(K))GOTO14
13      CONTINUE
        GOTO999
14      DO 15 K=1,NPRIOR
        Z(K)=ZTEST(K)
15      CONTINUE
        M=1
        RETURN
12      M=0
        RETURN
999     PRINT 600
        PRINT 601
        DO 998 K=1,NOBJ
        PRINT 602,K,Y(K)
998     CONTINUE
        PRINT 603
        DO 997 K=1,NVAR
        PRINT 604,K,X(K)
997     CONTINUE
        STOP
600     FORMAT(1H1,25X,'SEARCH ALGORITHM IS COMPLETED -- TOLERANCES FOR IM
       $PROVED SOLUTIONS SATISFIED')
601     FORMAT(1H1,25X,'OPTIMAL OBJECTIVE FUNCTION VALUES')
602     FORMAT(1H ,25X,'Y(',I2,')   =    ',F13.6)
603     FORMAT(1H1,25X,'OPTIMAL DECISION VARIABLE VALUES')
604     FORMAT(1H ,25X,'X(',I2,')   =    ',F13.6)
        END
        SUBROUTINE YVALUE
        INTEGER FSTART,STEP,FREQ
        COMPLEX ZW,ZPR,KAY(9),ZLINE,ZWEB,ZRAD,ZHEAD,ZMECH,CCOS,CSIN,ZLOAD,
       $J,S(3),LB,LF,ADMIT,ARG,KL(9),ZTAIL,ZPRESS,ZWATER,XTAN,XSIN
        REAL L(9),LFRONT,LBACK,SINH,COSH,TANH
        DIMENSION RHO(9),C(9),A(9),ETA(9),F(3),Y(12),Z(9),COST(9)
        COMMON/COMM12/RHO,C,L,A,ETA
        COMMON/COMM13/TURNS,CAP,ZW,ZPR,STEP,FY,COST
        COMMON/COMM07/Y
        SINH(X)= (EXP(X)-EXP(-X))/2
        COSH(X)= (EXP(X)+EXP(-X))/2
        TANH(X)= (EXP(X)-EXP(-X))/(EXP(X)+EXP(-X))
        XTAN(ARG)=(TAN(REAL(ARG))+J*TANH(AIMAG(ARG)))/(1-J*TAN(REAL(ARG))
       $*TANH(AIMAG(ARG)))
        XSIN(ARG)=SIN(REAL(ARG))*COSH(AIMAG(ARG))+J*COS(REAL(ARG))*SINH(
       $AIMAG(ARG))
        ZLINE(ZLOAD,ZMAT,ARG)= ZMAT*(ZLOAD+J*ZMAT*XTAN(ARG))/(ZMAT + J*ZLO
       $AD*XTAN(ARG))
        J=(0.0,1.0)
        PI= 3.14159
        DO 3 I=1,8
        Z(I)=RHO(I)*C(I)*A(I)
3       KAY(I)=(2*PI*L(I)*(1-J*ETA(I)))/C(I)
        ZWATER=ZW*A(1)
        ZPRESS=ZPR*A(8)
        FSTART=IFIX(FY)-500
```

```
      S(1)=(0.0,0.0)
      S(2)=(0.0,5.0E 04)
      S(3)=(0.0,-5.0E 04)
      F(1)=F(2)=F(3)=0.0
C BEGIN FREQUENCY LOOP
   18 FREQ=FSTART-STEP
   17 CONTINUE
      FREQ=FREQ+STEP
      DO 4 I=1,8
    4 KL(I)=FREQ*KAY(I)
      ZTAIL=ZLINE(ZLINE((0.0,0.0),Z(6),KL(6)),Z(5),KL(5))
      ZWEB= ZLINE(ZPRESS,Z(8),KL(8))
      ZRAD=ZLINE(ZLINE(ZWATER,Z(7),KL(7)),Z(1),KL(1))
      ZWEB=ZWEB+ZRAD
      ZHEAD=ZLINE(ZLINE(ZWEB,Z(2),KL(2)),Z(3),KL(3))
      ZMECH= -J*Z(4)/XSIN(KL(4)) +(ZHEAD+J*Z(4)* XTAN(KL(4)/2))*(ZTAIL+J
     1*Z(4)*XTAN(KL(4)/2))/(ZHEAD+ZTAIL+2*J*Z(4)*XTAN(KL(4)/2))
      ADMIT= J*2*PI*FREQ*CAP +TURNS/ZMECH
C COMPARE VALUES FOR MAX,MIN ON REAL AND IMAGINARY IMPEDANCE AXES
      IF(REAL(ADMIT).GE.REAL(S(1)))GO TO 12
      IF(FREQ.EQ.FSTART+STEP)GO TO 15
      GO TO 13
   12 S(1)=ADMIT
      F(1)=FREQ
      LF=ZHEAD
      LB=ZTAIL
      IF (AIMAG(ADMIT).LE.AIMAG(S(3))) GO TO 14
      S(3)=ADMIT
      F(3)=FREQ
      GO TO 17
   14 IF(FREQ.EQ.FSTART+STEP) GO TO 15
      GO TO 17
   13 IF(AIMAG(ADMIT).GE.AIMAG(S(2))) GO TO 19
      S(2)=ADMIT
      F(2)=FREQ
      GO TO 17
   15 FSTART=FSTART-50
      S(3)=(0.0,-5.0E04)
      S(1)=(0.0,0.0)
      F(1)=F(3)=0.0
      GO TO 18
   19 CONTINUE
C END OF FREQ LOOP     NOW VALUES FOR Y ARE CALCULATED
      CC=AIMAG(S(1))/(2*PI*F(1))
      DY=CABS(S(2)-S(3))
      IF(REAL(LB).EQ.0..AND.AIMAG(LB).EQ.0.) GO TO 40
      LBACK=(L(4)/REAL(KL(4)))*ATAN(ABS(REAL(J*Z(4)/LB)))
      GO TO 41
   40 LBACK=L(4)/2.0
   41 IF(REAL(LF).EQ.0..AND.AIMAG(LF).EQ.0.) GO TO 42
      LFRONT=(L(4)/REAL(KL(4)))*ATAN(ABS(REAL(J*Z(4)/LF)))
      GO TO 43
   42 LFRONT=L(4)/2.0
   43 CONTINUE
      Y(1)=F(1)
      Y(2)=LBACK/LFRONT
      Y(3)=F(1)/(F(2)-F(3))
```

```
      CMOT=DY/(Y(3)*2*PI*F(1))
      Y(4)=SQRT(ABS(CMOT/(CMOT+CC)))
      Y(5)=L(1)+L(2)+L(3)+L(4)+L(5)+L(6)
      Y(6)=Z(1)*L(1)/C(1)+Z(2)*L(2)/C(2)+Z(3)*L(3)/C(3)+Z(4)*L(4)/C(4)+
     1Z(5)*L(5)/C(5)+Z(6)*L(6)/C(6)
      Y(7)=CABS(S(1))
      Y(8)=ATAN(AIMAG(S(1))/REAL(S(1)))
      Y(10)=LBACK/L(4)
      Y(11)=LFRONT/L(4)
      Y(9)=0.0
      DO 20 I=1,8
   20 Y(9)=Y(9) + COST(I)*A(I)*L(I)
      FY= F(3) + 500.
      RETURN
      END
C***********************************************************************
      SUBROUTINE VARIB
      REAL L(9)
      DIMENSION RHO(9),C(9),A(9),ETA(9),X(12)
      COMMON/COMM12/RHO,C,L,A,ETA
      COMMON/COMM02/X
C USER SUPPLIED VARIABLES ARE LISTED HERE
      L(6)=X(1)
      L(5)=X(2)
      L(3)=X(3)
      L(2)=X(4)
      L(1)=X(5)
      RETURN
      END
```

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